# Hierarchical Bayesian modelling of exposure at default for revolving credit facilities

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Credit Scoring & Credit Control XVI

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# Outline

1. Some background on the problem.
2. The data used for model fitting/testing. We use Barclays' view of GCD data.
3. Model specifications. Reparameterisation, sampling from posterior and predictive distributions.
4. Results. Model selection. Comparison with Tobit model.
5. Discussion and future work.

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# Introduction

- Exposure at default (EAD) is an estimate of the exposure against a facility upon default of an obligor.
- EAD feeds into the calculation of RWA.
- CRR refers to credit conversion factors (CCF), the proportion of available headroom drawn between observation and default.
- By definition we have $EAD = B_0 + CCF(L_0 - B_0)$, where $B_0$ and $L_0$ are the balance and limit at observation.
- Banks with advanced permissions can model their own CCF (CRR Article 166).
- The time horizon for EAD estimates is one year (SS11/13 14.11).
- Estimates for EAD must be no less than current drawings, hence $CCF \geq 0$.
- Modelling can be performed on EAD directly or through CCF.

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# Our Approach

- Modelling CCF directly is challenging due to instability caused by small/zero headroom, i.e. when $L_0 - B_0 < \epsilon$.
- EAD typically exhibits very long tails. Modelling Usage at Default, $UAD = EAD/L_0$, means estimates are default weighted and not exposure weighted. Referred to as the *momentum* approach.
- We consider a zero-one inflated beta regression model for CCF.
- We shift and scale the beta-distributed random variable to obtain a parametric distribution for UAD.
- This enables us to model UAD directly but also constrains the implicit modelled values for CCF to the unit interval, ensuring that $UAD \geq B_0/L_0 :=$ Usage Before Default (UBD).
- We introduce a margin of conservatism by centering the prior distribution for mean CCF on the standardised value of 75%.
- Models are implemented in the probabilistic programming language `stan` which provides efficient MCMC sampling from the joint posterior distribution of the model parameters.

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# Data Sources

- We source Corp & FI data from GCD.
- We clean the data keeping only facilities where:
  - there is positive headroom,
  - balance at default is greater than one year prior,
  - balance at default does not exceed limit one year prior.
- This leaves 521 facilities for modelling (summarised in Table →). Note Long tails.
- We source GDP data for different regions (description below ↓).

Table: Observed summary statistics for Balance 1ypd, CCF and EAD

| | Balance 1ypd | CCF | EAD |
|---|---|---|---|
| Min. | 0 | 0.000 | 0 |
| 1st Qu. | 0 | 0.032 | 2031 |
| Median | 49 | 0.516 | 188518 |
| Mean | 3336193 | 0.501 | 5094438 |
| 3rd Qu. | 702088 | 0.997 | 2755172 |
| Max. | 125296671 | 1.000 | 143903572 |

Table: Description of macro-economic variables used for modelling

| Region | Source | Description |
|---|---|---|
| UK | ONS | U.K.: Gross Domestic Product, Current Prices (SA, Y/Y % chg) |
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# We calibrate the model to UAD

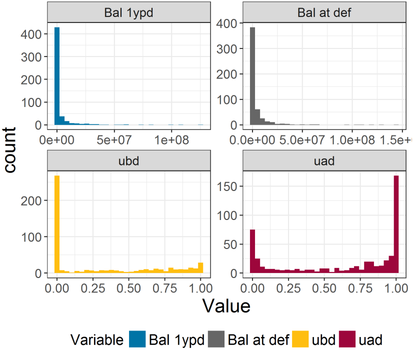


Figure: Histograms of Balance at one year prior to default, balance at default, UBD and UAD

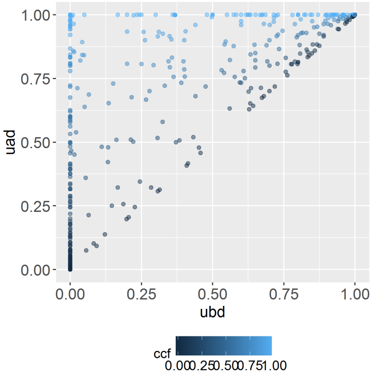


Figure: UAD against UBD. Lighter coloured circles indicate higher CCF.

# Potential Model Drivers

- We define classes for CCF: (0, 0.5], (0.5, 1), one and zero.
- The table shows average UBD, UAD and GDP by CCF class.
- Data indicates that UBD is on average higher for larger values of CCF.
- Mean GDP is on average higher where CCF is one compared to when CCF is zero.

Table: Mean UBD, UAD and GBP by CCF class: continuous in (0, 0.5], (0.5, 1), one or zero

| CCF class | Count | mean UBD | mean UAD | mean GDP |
|---|---|---|---|---|
| (0,0.5] | 164 | 0.216 | 0.344 | 1.940 |
| (0.5,1) | 143 | 0.373 | 0.867 | 2.302 |
| one | 120 | 0.340 | 1.000 | 1.296 |
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# Model Parameterisation - Mixture of Beta and Binomial

- We follow Ospina and Ferrari (2012) and induce zero-one inflation via a mixture of Bernoulli and beta distributions.
- Letting $y = CCF$, the density for $y$ is

$$f_{CCF}(y; \mu, \phi, \pi, \theta) = \pi Ber(\theta) + (1-\pi) Beta(\mu, \phi)$$

- Parameters $\pi$ and $\theta$ represent

$$\begin{aligned} \pi &= P(y \in \{0,1\}) \\ \theta &= P(y = 1 | y \in \{0,1\}) \end{aligned}$$

- These are related to $p_0 = P(y=0)$ and $p_1 = P(y=1)$ as follows:

$$\begin{aligned} p_0 &= P(y=0|y \in \{0,1\})P(y \in \{0,1\}) + P(y=0|y \in (0,1))P(y \in (0,1)) \\ &= (1-\theta)\pi \\ p_1 &= P(y=1|y \in \{0,1\})P(y \in \{0,1\}) + P(y=1|y \in (0,1))P(y \in (0,1)) \\ &= \theta\pi. \end{aligned}$$

- We recover $\pi$ if we compute $p_0 + p_1$.

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- We follow Ospina and Ferrari (2012) and induce zero-one inflation via a mixture of Bernoulli and beta distributions.
- Letting $y = CCF$, the density for $y$ is

$$f_{CCF}(y; \mu, \phi, \pi, \theta) = \pi Ber(\theta) + (1 - \pi) Beta(\mu, \phi)$$

- Parameters $\pi$ and $\theta$ represent

$$\begin{aligned} \pi &= P(y \in \{0, 1\}) \\ \theta &= P(y = 1 | y \in \{0, 1\}) \end{aligned}$$

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# Model Parameterisation - Shifted and scaled beta distribution

- Suppose $X$ has a beta$(\mu, \phi)$ distribution with $E(X) = \mu \in (0,1)$ and $Var(X) = \mu(1-\mu)/(\phi+1) > 0$.

- Let $Y = a + (b-a)X$. The support of $Y$ is $(a,b)$ and the density of $Y$ is

$$f_Y(y;\mu,\phi,a,b) = \begin{cases} \frac{1}{B(\mu\phi,(1-\mu)\phi-1)} \frac{(y-a)^{\mu\phi-1}(b-y)^{(1-\mu)\phi-1}}{(b-a)^{\phi-1}}, & y \in (a, a+b) \\ 0 & \text{otherwise} \end{cases}$$

- We use notation $Y \sim Beta_{(a,b)}(\mu,\phi)$ to indicate the support of $Y$ is $(a,b)$

- If $CCF \sim Beta_{(0,1)}(\mu,\phi)$, then

$$UAD = UBD + CCF(1-UBD) \sim Beta_{(ubd,1)}(\mu,\phi).$$

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# Marginal Expectations

- The parameter $\mu$ represents the conditional expectation of $CCF$ given $CCF$ is in the open unit interval.

$$\mu = E\{CCF|CCF \in (0,1)\}$$

- The marginal expectation of CCF is

$$\begin{aligned} E(CCF) &= E(CCF|CCF=0)P(CCF=0) + \\ &\quad E(CCF|CCF \in (0,1))P(CCF \in (0,1)) + \\ &\quad E(CCF|CCF=1)P(CCF=1) \\ &= \mu(1-\pi) + \theta\pi \end{aligned}$$

- Conditional mean of UAD is

$$E\{UAD|UAD \in (UBD,1)\} = UBD + (1-UBD)\mu$$

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# Likelihood and Prior Distributions

- We regress each model parameter on a set of covariates via a suitable link function and give vague prior distributions to the regression coefficients:

$$\begin{aligned}
\mu &= \text{inv_logit}(x_i^{\mu}\beta^{\mu}), \quad \beta^{\mu} \sim N(0, 10^4 I) \\
\phi &= \exp(x_i^{\phi}\beta^{\phi}), \quad \beta^{\phi} \sim N(0, 10^4 I) \\
\pi &= \text{inv_logit}(x_i^{\pi}\beta^{\pi}), \quad \beta^{\pi} \sim N(0, 10^4 I) \\
\theta &= \text{inv_logit}(x_i^{\theta}\beta^{\theta}), \quad \beta^{\theta} \sim N(0, 10^4 I)
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- The complete set of model parameters is $\Omega = (\mu, \phi, \pi, \theta, \beta)$ where $\beta = (\beta^{\mu}, \beta^{\phi}, \beta^{\pi}, \beta^{\theta})$.

- The likelihood function can be written as

$$\begin{aligned}
L(\Omega; UAD, UBD) &= \prod_{i=1}^{n} \{(1-\theta_i)\pi_i\}^{\mathbb{I}(UAD_i = UBD_i)} (\theta_i \pi_i)^{\mathbb{I}(UAD_i = 1)} \times \\
&\quad \{(1-\pi) Beta_{(UBD_i, 1)}(\mu_i, \phi_i)\}^{\mathbb{I}(UBD_i < UAD_i < 1)}.
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- Strictly, the likelihood is a function of $\mu, \phi, \pi, \theta$ only through $\beta$ via the deterministic link functions. However, on occasion we will model one or more of $\mu, \phi, \pi, \theta$ directly.


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# Posterior Distributions

- The posterior distribution for $\Omega$ is given by

$$f(\Omega|UAD, UBD) \quad \propto \quad L(\Omega; UAD, UBD) f(\Omega)$$

- Given $N$ post burn-in samples from $f(\Omega|UAD, UBD)$ the estimates for $CCF_i$ and $UAD_i$ are calculated as follows:

$$\widehat{CCF_i} = \frac{1}{N}\sum_{t=1}^{N} \mu_i^{(t)}(1-\pi_i^{(t)}) + \theta_i^{(t)}\pi_i^{(t)}$$

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- We can obtain 95% credible interval from the samples $CCF^{(t)}$ and $UAD^{(t)}$.
- For readability, change notation to let $y = UAD$ and $x = UBD$.
- Prediction for out-of-sample data $y_0 = UAD_0$, requires sampling from the posterior predictive distribution.

$$f(y_0|x_0, y, x) = \int f(y_0|x_0, \Omega) f(\Omega|y, x) d\Omega$$

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$$\widehat{CCF_i} = \frac{1}{N}\sum_{t=1}^{N} \mu_i^{(t)}(1-\pi_i^{(t)}) + \theta_i^{(t)}\pi_i^{(t)}$$

$$\widehat{UAD_i} = \frac{1}{N}\sum_{t=1}^{N} UBD_i + (1-UBD_i)\{\mu_i^{(t)}(1-\pi_i^{(t)}) + \theta_i^{(t)}\pi_i^{(t)}\}$$

- We can obtain 95% credible interval from the samples $CCF^{(t)}$ and $UAD^{(t)}$.
- For readability, change notation to let $y = UAD$ and $x = UBD$.
- Prediction for out-of-sample data $y_0 = UAD_0$, requires sampling from the posterior predictive distribution.

$$f(y_0|x_0, y, x) = \int f(y_0|x_0, \Omega)f(\Omega|y, x)d\Omega$$

- If $\Omega^{(t)} \sim f(\Omega|y, x)$ then $y_0^{(t)} \sim f(y_0|x_0, \Omega^{(t)})$ is a draw from the posterior predictive distribution $f(y_0|x_0, y, x)$.



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# Candidate models

- We look at different regression models for the parameters using available covariates UBD and GDP.

- Where we do not have a regression model for a parameter, we estimate it directly rather than through $\beta_0$. For example, we do not place a regression model on $\pi$, instead the prior is $\pi \sim U(0,1)$.

- We use the notation $\mu(ubd, gdp)$ to indicate that we model $\mu = \text{inv\_logit}(\beta_0 + \beta_1 ubd + \beta_2 gdp)$.

- A benchmark model for UAD is the Tobit model:
  - The Tobit model assumes a normal distribution but only observed for the unit interval. Point masses at zero and one.
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# Model Selection Results

- Models are fit to 80% of the data, holding out 20% for testing.
- Inference is based on 3000 post burn-in samples generated from three parallel MCMC chains.
- Model performance is judged by WAIC (widely-applicable IC), which is appropriate for Bayesian analysis (Watanabe, 2010), and prediction RMSE.
- The best performing model is M7 with $\mu(ubd), \theta(gdp)$.
- The best Tobit model has $\mu(ubd, gdp), \sigma^2(ubd, gdp)$ where $\sigma^2$ is modelled via a log-link.

Table: WAIC and RMSE for a selection of models

| Model | Description | WAIC | RMSE |
|---|---|---|---|
| M1 | $\mu(ubd)$ | 892.1 | 0.341 |
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The RMSE for the best Tobit model is 0.346.



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# Parameter Estimates for Best Model

Table: Parameter estimates for Model M7: $\mu(ubd), \theta(gdp)$

| parameter | mean | sd | 2.5% | 97.5% | $\hat{R}$ |
|---|---|---|---|---|---|
| $\mu(\beta_0)$ | -0.066 | 0.079 | -0.221 | 0.096 | 0.999 |
| $\mu(\beta_1)$ | 0.326 | 0.092 | 0.145 | 0.513 | 0.999 |
| $\phi$ | 1.066 | 0.048 | 0.977 | 1.170 | 0.999 |
| $\pi$ | 0.398 | 0.025 | 0.353 | 0.448 | 0.999 |
| $\theta(\beta_0)$ | 0.426 | 0.170 | 0.101 | 0.756 | 1.002 |
| $\theta(\beta_1)$ | -0.578 | 0.166 | -0.921 | -0.281 | 0.999 |

- $\mu(\beta_1)$ indicates a significant correlation between the logit of conditional expectation of CCF and UBD.
- $\theta(\beta_1)$ indicates that GDP is a driver for $\theta$ and hence $p_0$ and $p_1$.
- $\hat{R}$ is an estimate of the potential scale reduction factor which is a well-known convergence statistic. Values below 1.1 are used to indicate convergence of the Markov chain to the posterior distribution of the parameter. (Gelman and Rubin, 1992).

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# Box plots of facility-level effects

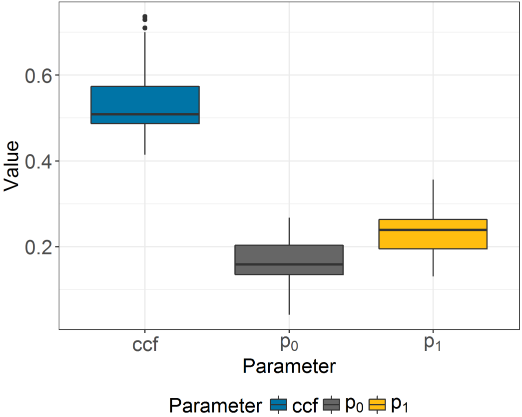


Figure: Boxplots of parameter estimates for $CCF$, $p_0$ and $p_1$ for model M7

- $p_0$ and $p_1$ vary over $i$ due to dependence on $\theta$.

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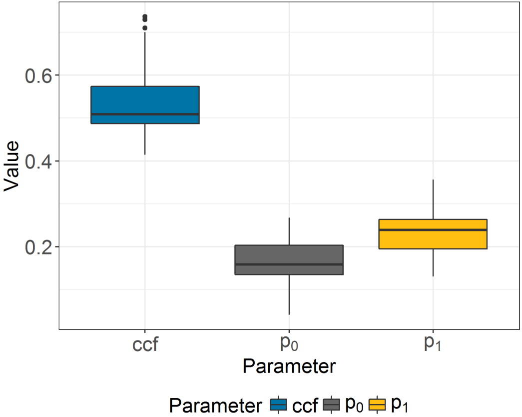


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# Conservative Prior Distribution

- We embed conservatism by adjusting the prior distribution for the conditional mean of CCF, $\mu$.
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- Re-arranging gives prior standard deviation $\sqrt{v_0} = m_0/\Phi^{-1}(\alpha/2)$.
- All modelled CCF values receive an uplift. With 95% confidence ($\alpha = 5\%$) the average uplift is 1%, 99% confidence gives 1.5%.
- Strong prior belief does not strongly effect inference as the data overwhelms the prior.

Table: Parameter estimates Regulatory model

| parameter | mean | sd | 2.5% | 97.5% | $\hat{R}$ |
|---|---|---|---|---|---|
| $\mu(\beta_0)$ | -0.042 | 0.081 | -0.201 | 0.107 | 0.999 |
| $\mu(\beta_1)$ | 0.332 | 0.094 | 0.159 | 0.514 | 0.999 |
| $\phi$ | 1.062 | 0.049 | 0.968 | 1.160 | 0.999 |
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| parameter | mean | sd | 2.5% | 97.5% | $\hat{R}$ |
|---|---|---|---|---|---|
| $\mu(\beta_0)$ | -0.042 | 0.081 | -0.201 | 0.107 | 0.999 |
| $\mu(\beta_1)$ | 0.332 | 0.094 | 0.159 | 0.514 | 0.999 |
| $\phi$ | 1.062 | 0.049 | 0.968 | 1.160 | 0.999 |
| $\pi$ | 0.400 | 0.025 | 0.352 | 0.450 | 1.001 |
| $\theta(\beta_0)$ | 0.486 | 0.158 | 0.177 | 0.796 | 0.999 |
| $\theta(\beta_1)$ | -0.573 | 0.172 | -0.908 | -0.253 | 0.999 |

# Conservative Prior Distribution

- We embed conservatism by adjusting the prior distribution for the conditional mean of CCF, $\mu$.
- For the preferred model we have $\mu = \text{inv\_logit}(\beta_0 + \beta_1 UBD)$, where $\beta_k \sim N(m_k, v_k)$ for $k = 0, 1$.
- We set $m_0$ so that the prior mean is equal to the standardised value of 75% $m_0 = \text{logit}(0.75) = \log(0.75/(1-0.75)) = \log(3)$.
- We set $v_0$ so that the $\beta_0$ is significantly different from 0, $P(\beta_0 < 0) = \alpha/2$, where $\alpha$ is 1 - confidence level, the strength of belief in the standardised value.
- Re-arranging gives prior standard deviation $\sqrt{v_0} = m_0/\Phi^{-1}(\alpha/2)$.
- All modelled CCF values receive an uplift. With 95% confidence ($\alpha = 5\%$) the average uplift is 1%, 99% confidence gives 1.5%.
- Strong prior belief does not strongly effect inference as the data overwhelms the prior.

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BARCLAYS

# Summary

- We have described a fully Bayesian approach to modelling EAD.
- This properly accounts for uncertainty in parameter estimation when predicting new values.
- By shifting and scaling a beta distribution we build a parametric regression model for UAD.
- We show how the prior distribution can be used to embed conservatism into model predictions.

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# Future Work

- A simulation study demonstrating parameter recovery.
- Broader data collection for covariates.
- Full k-fold cross-validation.
- Wider model comparison.
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BARCLAYS

# stan

- stan is a probabilistic programming language providing efficient MCMC sampling from posterior distributions.
- There are 6 blocks: functions, data, parameters, model, transformed parameters and generated quantities.

```
functions {
  real generalised_beta_log_pdf(real y, real p, real q, real a) {
  return beta_lpdf(y | p, q) + (p - 1)*(log(y - a) - log(y)) - pow(1 - a, p + q - 1);
...
  }
data {
  int n;
  vector[n] x; //ubd
  vector<lower=0, upper=1>[n] y; //uad
...
}
parameters {
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# stan

```
transformed parameters {
  vector<lower=0>[n] p;        // scale 1 of beta(p,q)
  p = mu * phi;
...
}
model {
  Pi ~ uniform(0, 1); coef_theta ~ normal(0, sqrt(10000));
  coef_mu ~ normal(0, sqrt(10000)); phi ~ lognormal(0, sqrt(10000));
  for (i in 1:n) { // zero-one inflated beta likelihood
    if (y[i] == x[i]) {
      target += log(Pi) + log1m(theta[i]);
    } else if (y[i] == 1) {
      target += log(Pi) + log(theta[i]);
    } else {
      target += log1m(Pi) + generalised_beta_log_pdf(y[i] , p[i], q[i], x[i]);
    }
  }
}
generated quantities {
  vector[n] ccf_marg_exp;
  for (i in 1:n) ccf_marg_exp[i] = Pi*theta[i] + (1-Pi)*mu[i];
...
  }
```