The Extended Exogenous Maturity Vintage Model Across the Consumer Credit Lifecycle

Malwandla, M. C.[1,2]
Rajaratnam, K.[3]
Clark, A. E.[1]

1. Department of Statistical Sciences, University of Cape Town, Cape Town, South Africa
2. Standard Bank, Johannesburg, South Africa
3. Department of Finance & Tax and the African Collaboration for Quantitative Finance and Risk Research, University of Cape Town, Cape Town, South Africa
4. African Collaborative for Quantitative Finance and Risk Research, Cape Town, South Africa

Presented By: Musa Malwandla

Research Problem and Aim

- **Identifiability Problem in the EMV Model, and lack of behavioural data**
  - *Aim: bypass the identifiability problem*
  - *Aim: introduce behavioural data into model*
- **Credit risk models lack unification**
  - *Aim: show how model can be used across all areas of credit risk*
- **Lack micro-foundation for aggregation model, understanding of systemic risk**
  - *Aim: show how model aggregates to portfolio loss*
  - *Aim: provide general formula for asset correlation coefficient*
- **The problem of aggregating risk across portfolios**
  - *Aim: provide an approach for aggregating risk across portfolios*

Agenda

- **Model Specification**
  - Standard EMV Model
  - Extended EMV Model
  - Illustration: South African Portfolio
- **Application Areas**
  - Application and Behavioural Scorecard
  - Impairment Modelling
  - Stress Testing
  - *Capital Management*
- **Extension: Survival Analysis Decomposition**
- **Systemic Risk & Aggregation**
  - Understanding the LHP approximation
  - Understanding the Asset Correlation Coefficient
  - General Asset Correlation Coefficient Formula
  - Illustration: South African Portfolio
  - Understanding Diversifiable
- **Cross-Portfolio Aggregation**
  - A formula for cross-portfolio aggregation
  - Other uses of the formula
  - Illustration: South African Portfolio

Model Specification

Standard EMV Model

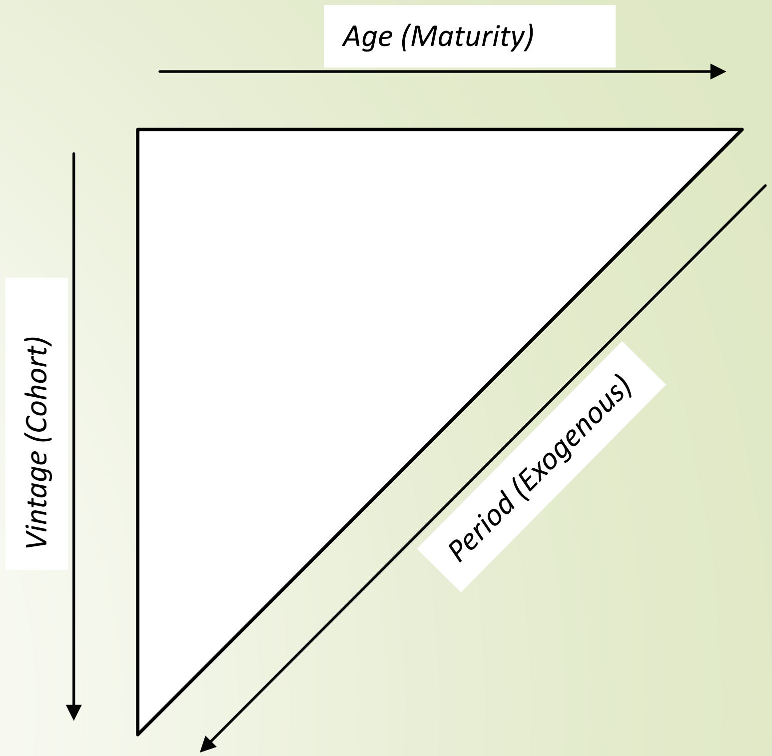
Extended EMV Model

Illustration: South African Portfolio

Standard EMV Model

$$p(t,s) = A_{t-s} + P_t + C_s$$

$$p(t,s) = M_{t-s} + E_t + V_s$$



APC Model

- Widely applied in epidemiology: Age Period Cohort Model
- Dimensional interpretation in mortality studies:
  - **Age**: effect of age on mortality (e.g, Gompertz Law, Accident Hump)
  - **Period**: effect of period on mortality (e.g., war, plague, cultural influence)
  - **Cohort**: effect of cohort on mortality (e.g., epigenetics, unnatural selection)

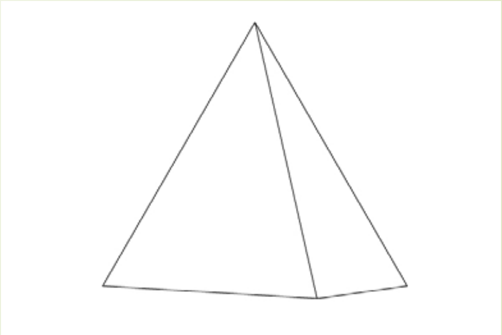
EMV Model

- **Exogenous**: mainly assumed to capture macroeconomic/policy environment
- **Maturity**: mainly captures risk by maturity and selective effects
- **Vintage**: mainly captures effect of acquisition policy (e.g. scorecard and cut-offs, target market)

Extended EMV Model: Specification

$$p(t,s,k) = \Phi(\alpha + M_{t-s} + E_t + \cancel{V_s} + B_k + A_k) \rightarrow \Phi\left(\mu + \sigma_M \ddot{M}_{t-s} + \sigma_E \ddot{E}_t + \sigma_V \cancel{\ddot{V}_s} + \sigma_B \ddot{B}_k + \sigma_B \ddot{A}_k\right)$$

- **Extend model beyond time dimensions, to behavioural dimensions**
  - Include behavioural score dimension / application score dimension…
- **Standardisation**
  - For convenience, standardise all component to mean = 0, std. dev = 1
  - Standard deviation of each component becomes parameter / significance measure
- **Identifiability**
  - Application scorecard attempts to capture same effect as vintage
  - Replace vintage with application scorecard
- **Link functions**
  - Logit, Probit, CLogLog
  - Prefer Probit: leads to Vasicek distribution for portfolio loss
  - Alternative is CLogLog: leads to Log-Log-Normal distribution for portfolio loss



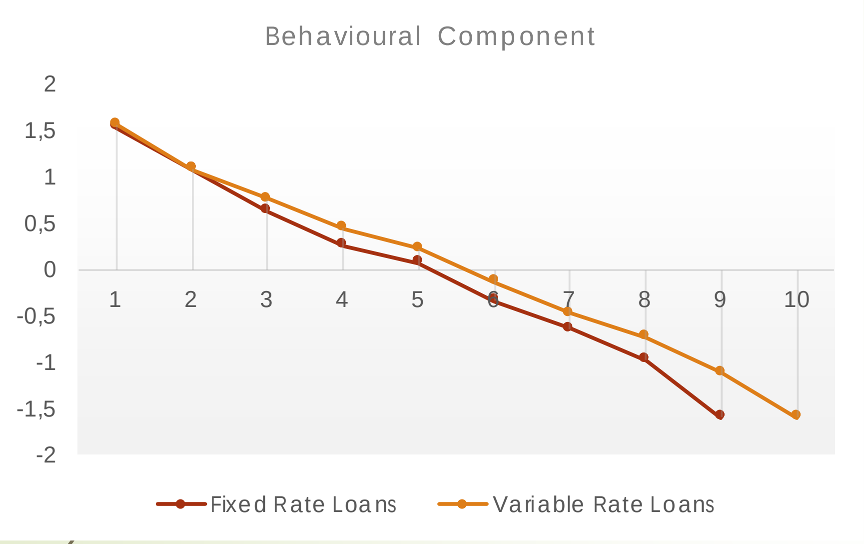
Fitting Illustration: Portfolio Description

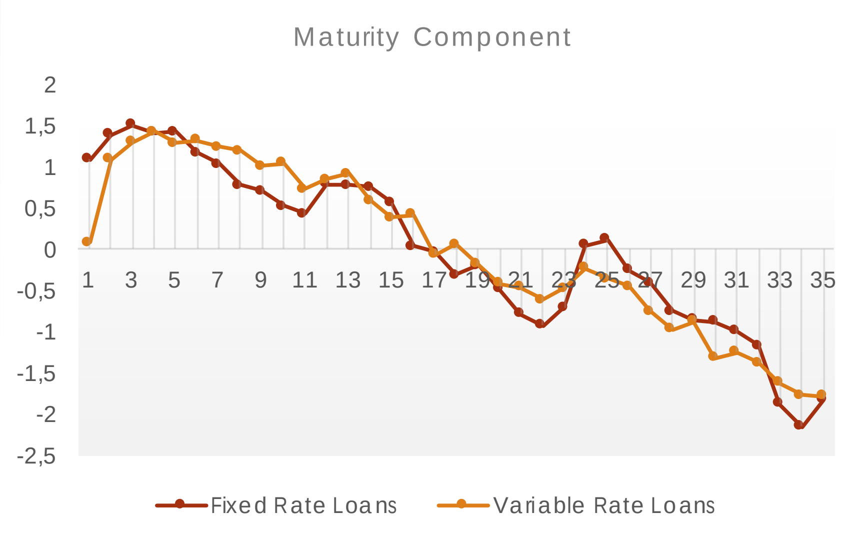
- South African consumer loan portfolio
  - **Fixed-Rate Loans**: interest rate fixed at the outset of the loan
  - **Variable-Rate Loans**: interest rate varies with central bank rate (Prime Overdraft Rate)
- Observations
  - 2.5m observations
  - September 2005 to June 2014
- Default Definition:
  - 90 Days Past Due, Distressed Restructure, Litigation, Write Off

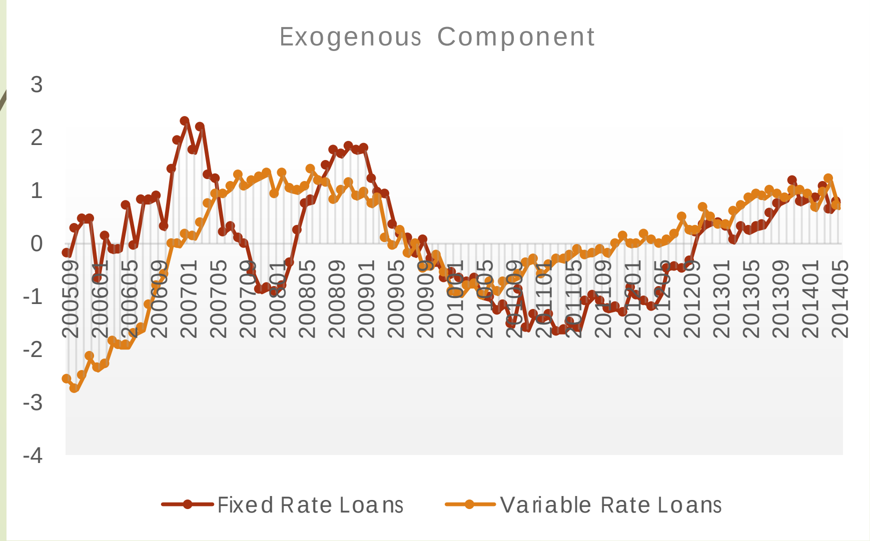
Fitting Illustration: Dimensionality

Model	Fixed Rate Loans (AIC)	Variable Rate Loans (AIC)	Model	Fixed Rate Loans (AIC)	Variable Rate Loans (AIC)
emvba	14023967	15088514	**mba**	14096703	15178034
emva	14024038	15088633	**ma**	14096783	15178132
emvb	14024061	15088682	**mb**	14096809	15178175
emba	14043687	15103096	**vba**	14145114	15202691
ema	14043758	15103204	**va**	14145183	15202810
emb	14043781	15103250	**vb**	14145203	15202857
evba	14044690	15104478	**ba**	14179438	15242878
eva	14044760	15104603	**a**	14179509	15242990
evb	14044782	15104653	**b**	14179531	15243036
mvba	14063296	15141550	**emv**	14867685	16650470
mva	14063372	15141656	**ev**	14884307	16667967
mvb	14063395	15141700	**em**	14903110	16675023
eba	14087834	15169005	**mv**	14907904	16701158
ea	14087897	15169131	**e**	14922954	16708235
eb	14087917	15169180	**v**	14963533	16750295
			m	15010479	16759464

Fitting Illustration: Components



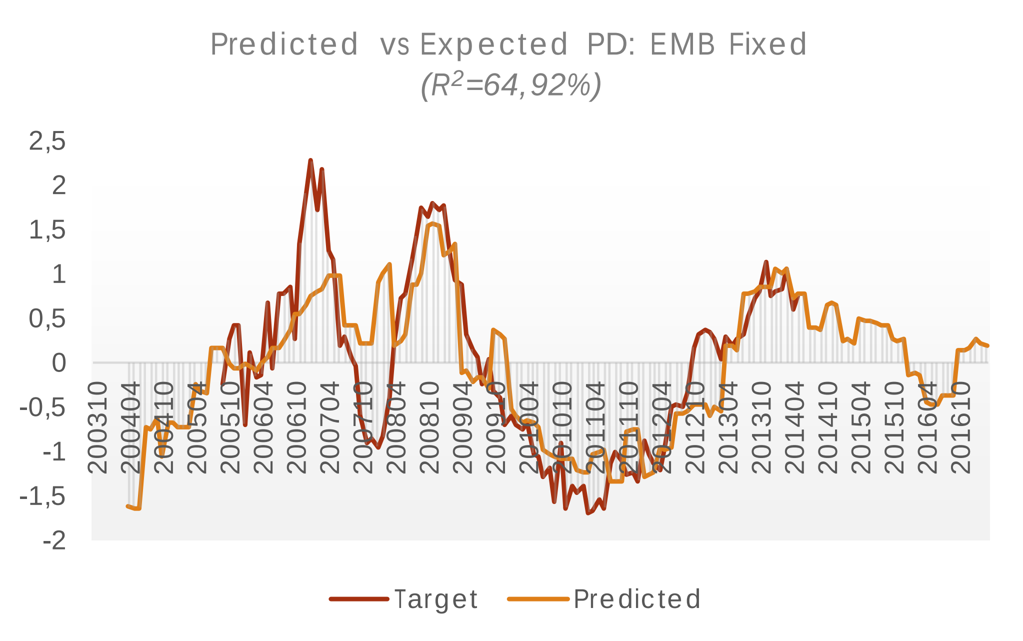


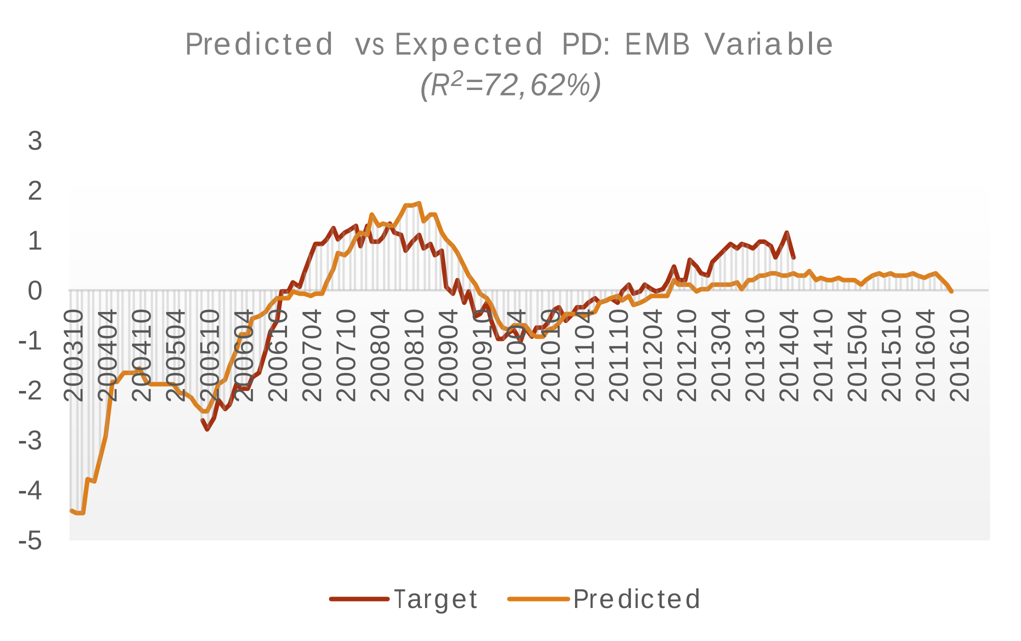


$$p(t,s,k) = \Phi\left(\mu + \sigma_M \ddot{M}_{t-s} + \sigma_E \ddot{E}_t + \sigma_B \ddot{B}_k\right)$$

Parameters	Fixed Rate Loans	Variable Rate Loans
α	-1.40292	-1.64782
μ_B	0.07809	0.18327
μ_E	-0.00048	-0.00034
μ_M	-0.00138	0.00495
μ	-1.32668	-1.45993
σ_B	0.37928	0.50060
σ_E	0.08707	0.10852
σ_M	0.11380	0.09351

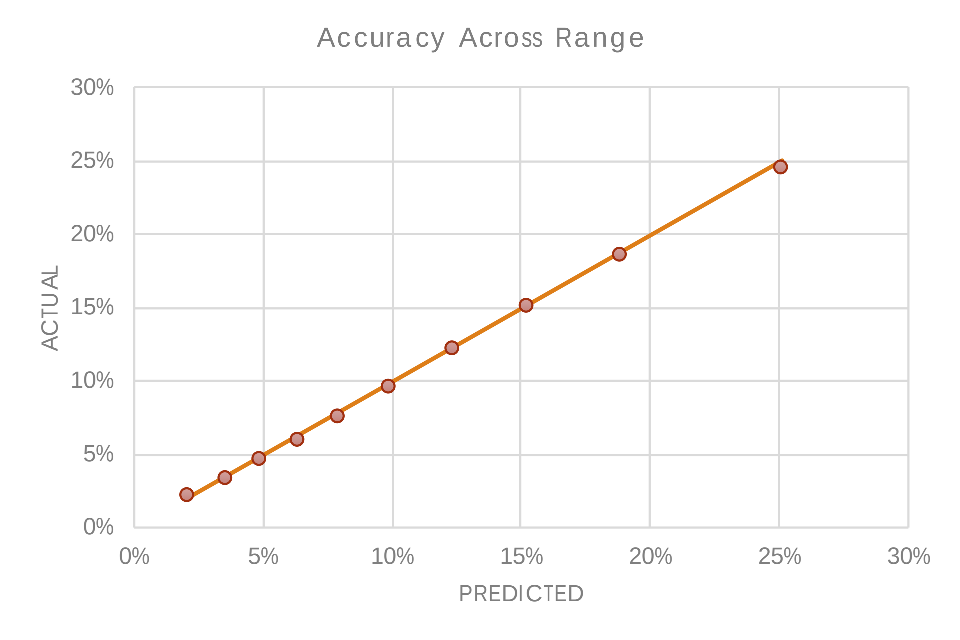
Fitting Illustration: Exogenous Model

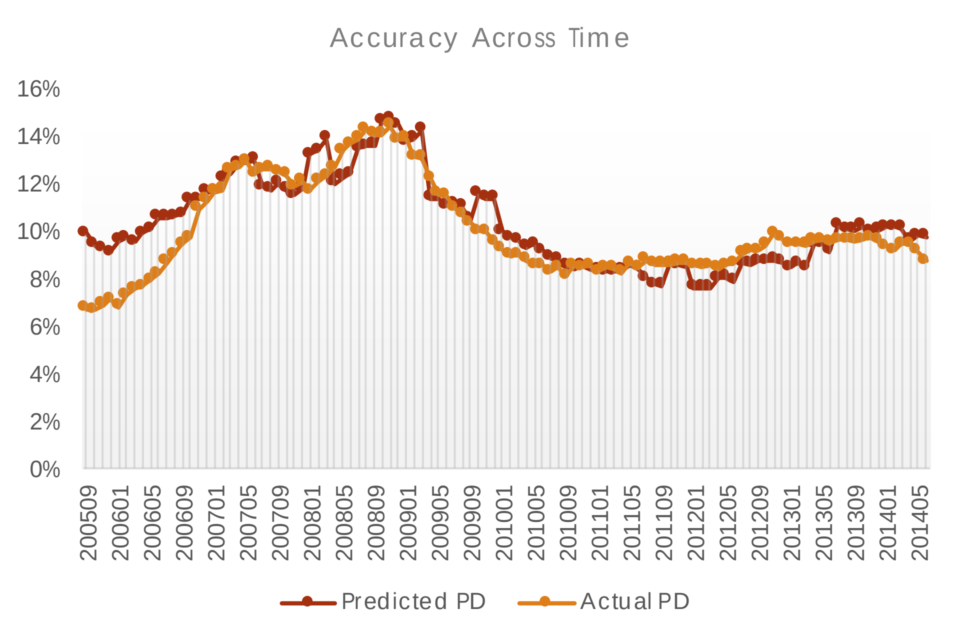




Fixed-Rate Model				Variable-Rate Model			
Variable	Estimate	P-Value	VIF	Variable	Estimate	Variable-Rate Model	VIF
Intercept	-11.0136	0.0590	0.00000	Intercept	-2.6211	0.0000	0.00000
Consumer Price Index	0.1775	0.0000	1.02611	Consumer Price Index	0.5030	0.0000	1
Ratio: Consumption to GDP	0.2624	0.0014	2.59047				
Ratio: Savings to GDP	-0.3510	0.0000	2.55178				

Fitting Illustration: Validation





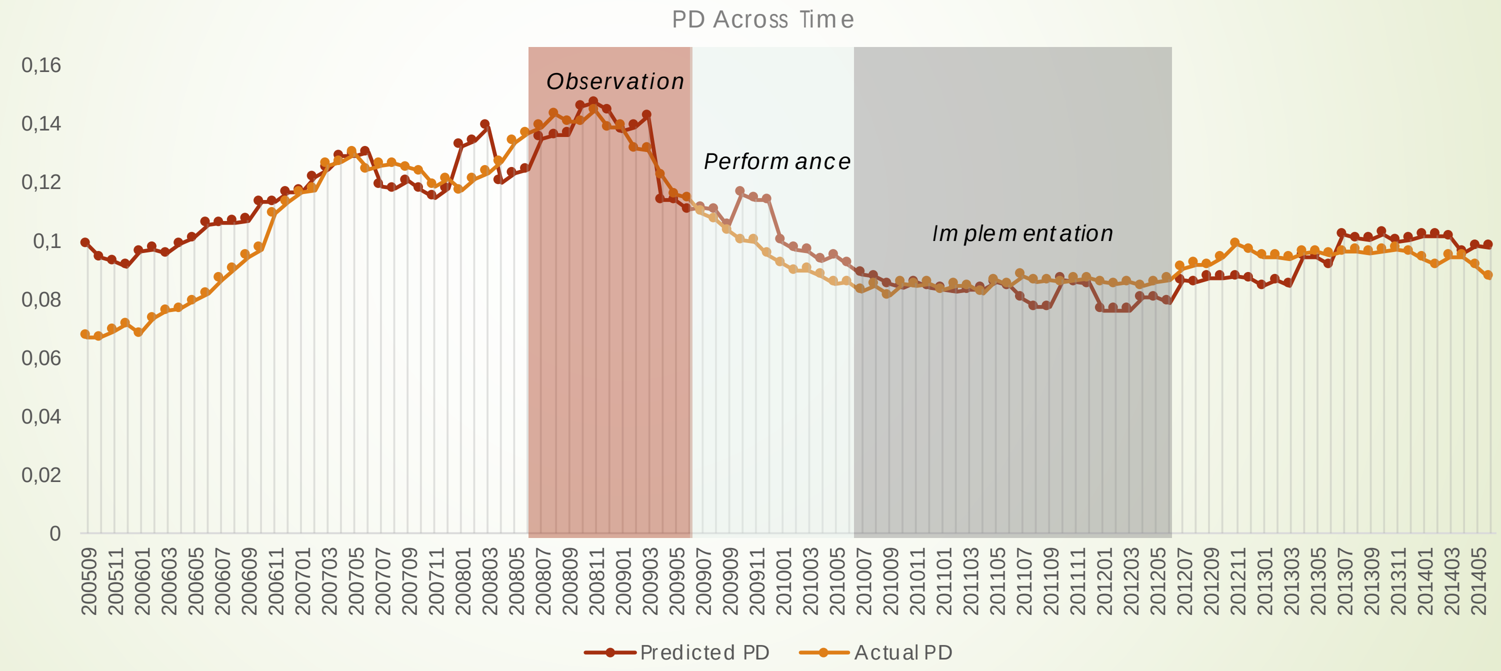
Gini Statistic	EMB	EMV
Fixed Rate Loans	37%	16%
Variable Rate Loans	47%	14%

Application Areas

- Application and Behavioural Scorecard
- Impairment Modelling
- Stress Testing
- Capital Management

Application Areas: Scorecards

- Linking risk scores to macroeconomic variables:
  - Dynamic score cut-offs
  - Reducing recalibration frequency



- IFRS 9 SICR swap-set problem can be resolved through EMBA model:
  - $p(t,s,k) = \Phi\left(E_t + R_{t-s,l,k}\right)$ where $R_{t,s,l,k} = V_l \alpha_{t-s} + B_k (1 - \alpha_{t-s})$ for monotonic α_{t-s}

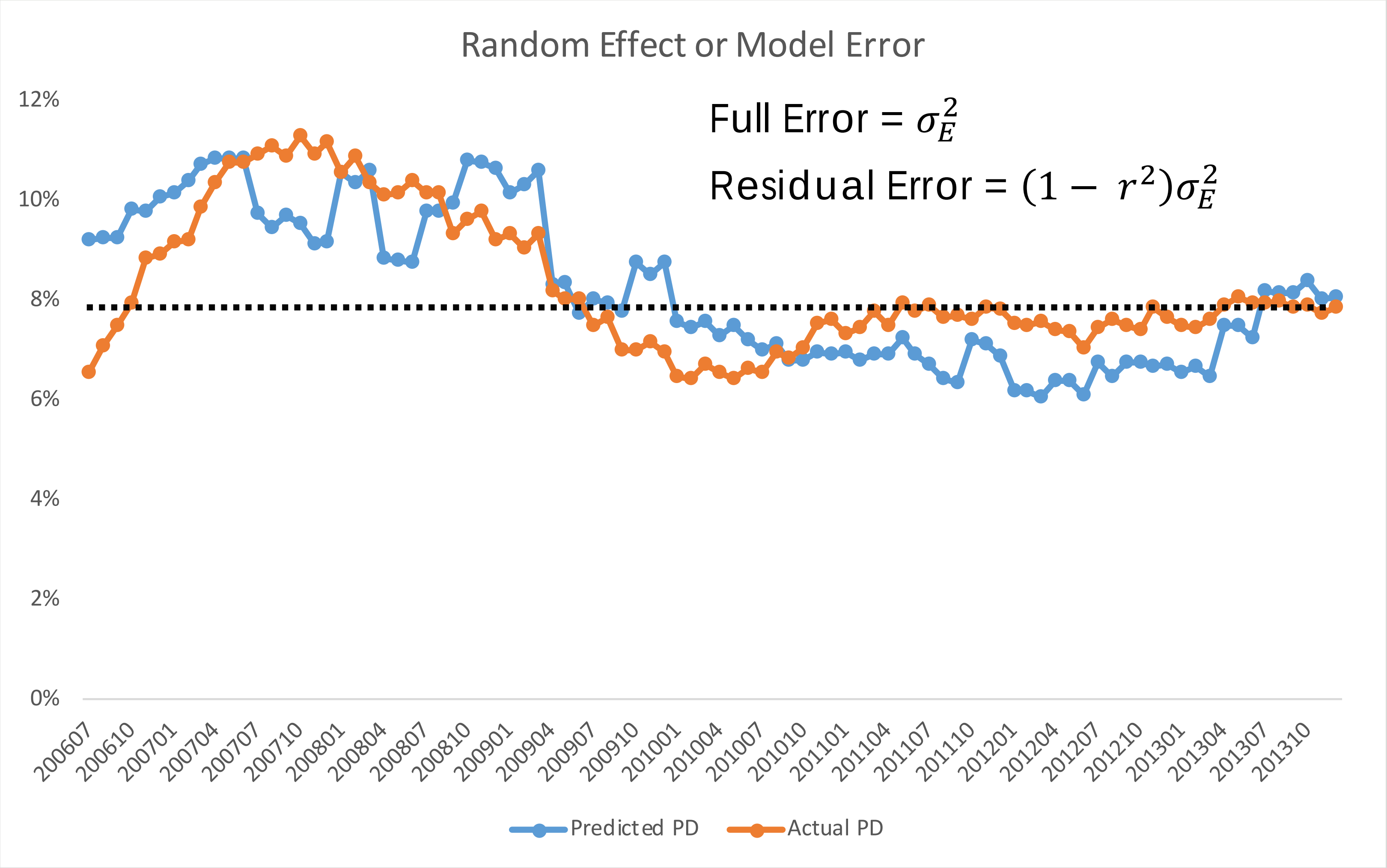
Application Areas: Impairments & Stress-Testing

- IFRS 9 Lifetime PD
  - Use Cumulative Incidence Function approach:

$$p_k(t, R_{t,s,l,k}) = \sum_{j=1}^{k} p_{12}(t, R_{t+j,s,l,k}) \times [1 - p_{12}(t, R_{t+j-1,s,l,k}) - q_{12}(t, R_{t+j-1,s,l,k})]$$

- Stress-Testing
  - Acquisition strategy using application score component.
  - Account management strategy, through behavioural score card.

Application Areas: Capital Management (Model Error)



Extended EMV Model: Summary

Application Area	Modelled Parameter	Point-in-Time / Forward-in-Time	Through-the-Cycle
Application / Behavioural Scoring / Stress Testing	Probability of Default; of Rolling (Collections); Write Off (LGD)	$\Phi\left(\frac{R_{t,s,l,k}}{\sqrt{1+(1-\ r^2)\sigma_E^2}}\right)$	$\Phi\left(\frac{K_{t,s,l,k}}{\sqrt{1+\sigma_E^2}}\right)$
IFRS 9 Impairment Modelling	Lifetime Probability of Default	Cumulative Incidence Function	N/A
Economic Capital	Value-at-Risk / Quantile Function	$\Phi\left[\sqrt{\frac{\rho}{1-\rho}}\Phi^{-1}(\alpha)+\sqrt{\frac{1}{1-\rho}}\Phi^{-1}(p_t)\right]$	
Economic Capital	Asset Correlation Coefficient	$\frac{\sigma_E^2(1-\ r^2)}{1+\sigma_E^2(1-\ r^2)}$	$\frac{\sigma_E^2}{1+\sigma_E^2}$
Economic Capital (Cross-Portfolio)	Cross-Portfolio Loss Distribution	$N(\mu,\sigma)$	$N(\mu,\sigma)$

- *Short hand*:
  - $R_{t,s,l,k} = \mu + \sigma_M \ddot{M}_{t-s} + \sigma_V \ddot{V}_l + \sigma_B \ddot{B}_t + \sigma_E \bar{E}_t$
  - $K_{t,s,l,k} = \mu + \sigma_M \ddot{M}_{t-s} + \sigma_V \ddot{V}_l + \sigma_B \ddot{B}_k$

Systemic Risk & Aggregation

Understanding the LHP approximation

Understanding the Asset Correlation Coefficient

General Asset Correlation Coefficient Formula

Illustration: South African Portfolio

Understanding Diversifiable

Understanding the LHP Approximation

?

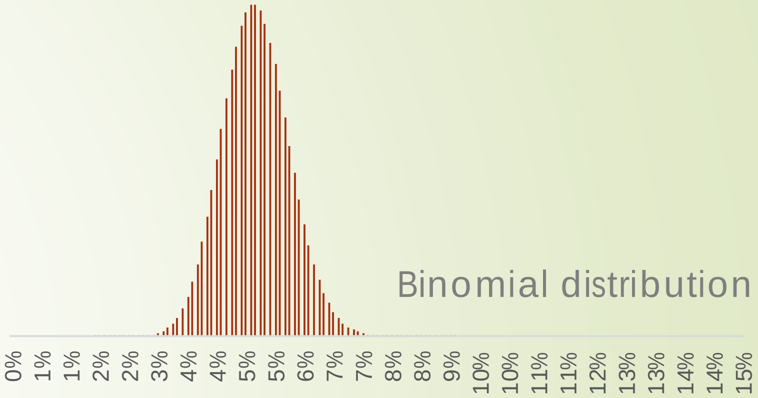
Unknown distribution

Condition on *e* →

?

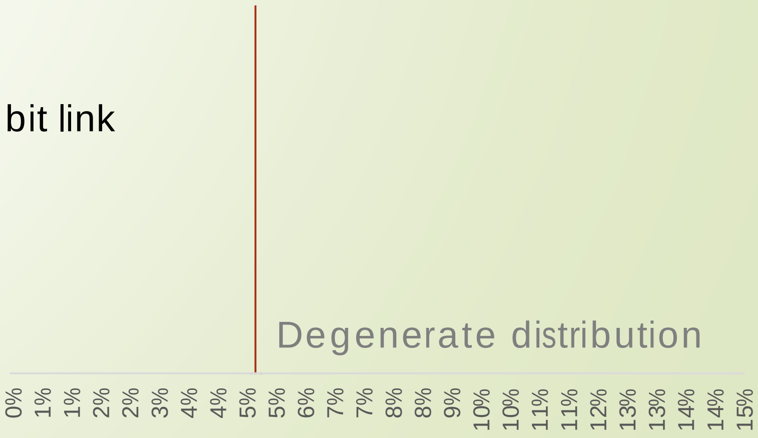
Unknown distribution

Assume homogeneity →



Binomial distribution

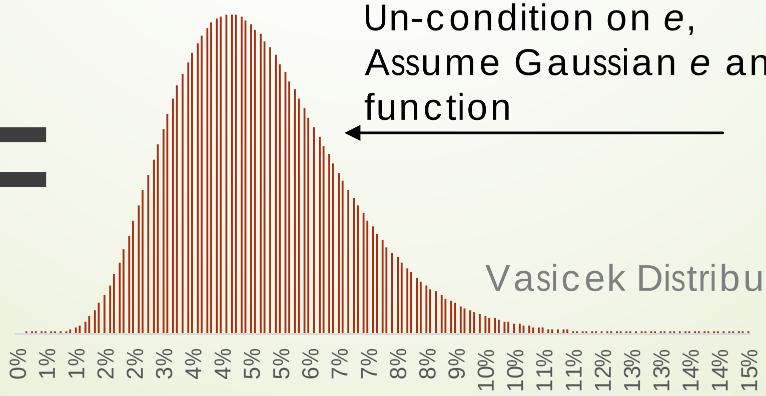
↓ Assume large portfolio



Degenerate distribution

← Un-condition on *e*, Assume Gaussian *e* and Probit link function

FYI: CLogLog link leads to log-log-normal distribution



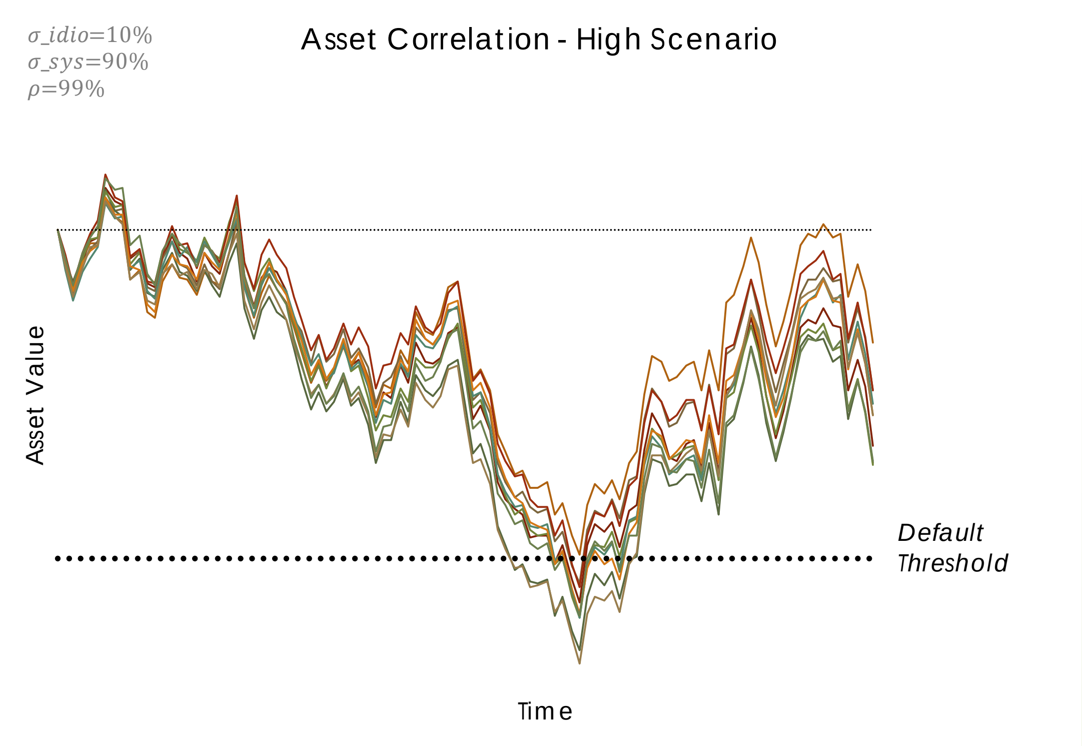
Vasicek Distribution

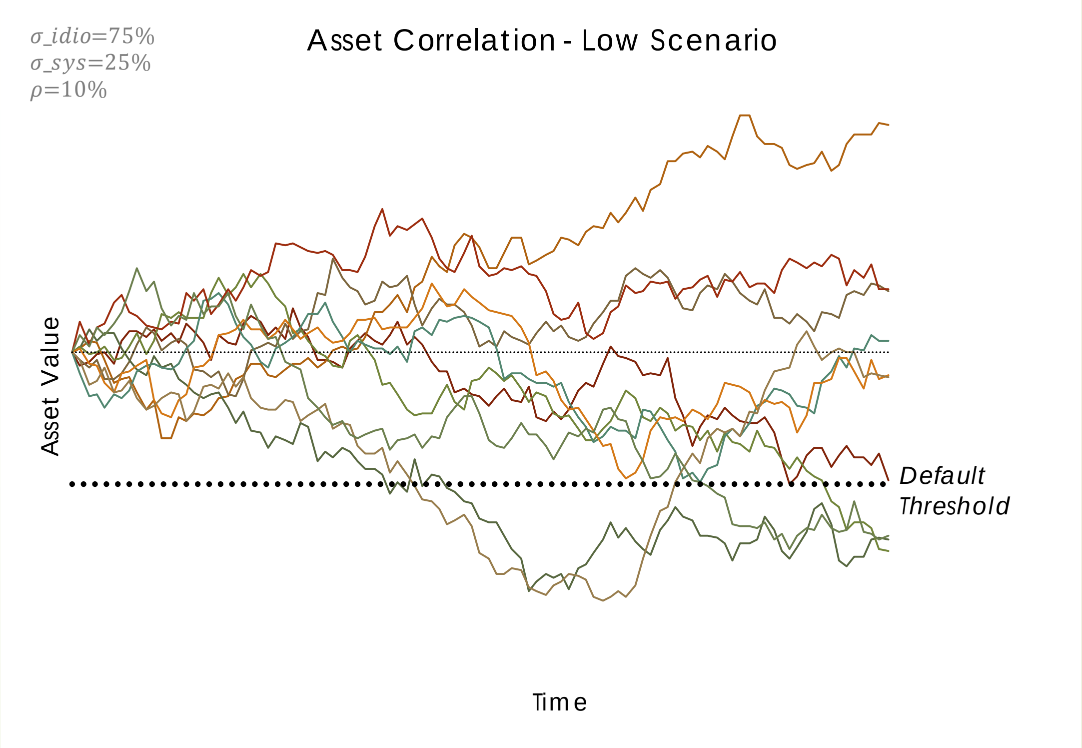
Vasicek Quantile Function

$$\Phi\left[\sqrt{\frac{\boldsymbol{\rho}}{1-\boldsymbol{\rho}}}\,\Phi^{-1}(\alpha)+\sqrt{\frac{1}{1-\boldsymbol{\rho}}}\,\Phi^{-1}(p_t)\right] =$$

Understanding the Asset Correlation Coefficient

- The asset correlation coefficient is the shape measure of the Vasicek distribution:
  - Influenced by level of correlation between portfolio, or
  - Influenced by level systemic risk relative to idiosyncratic risk



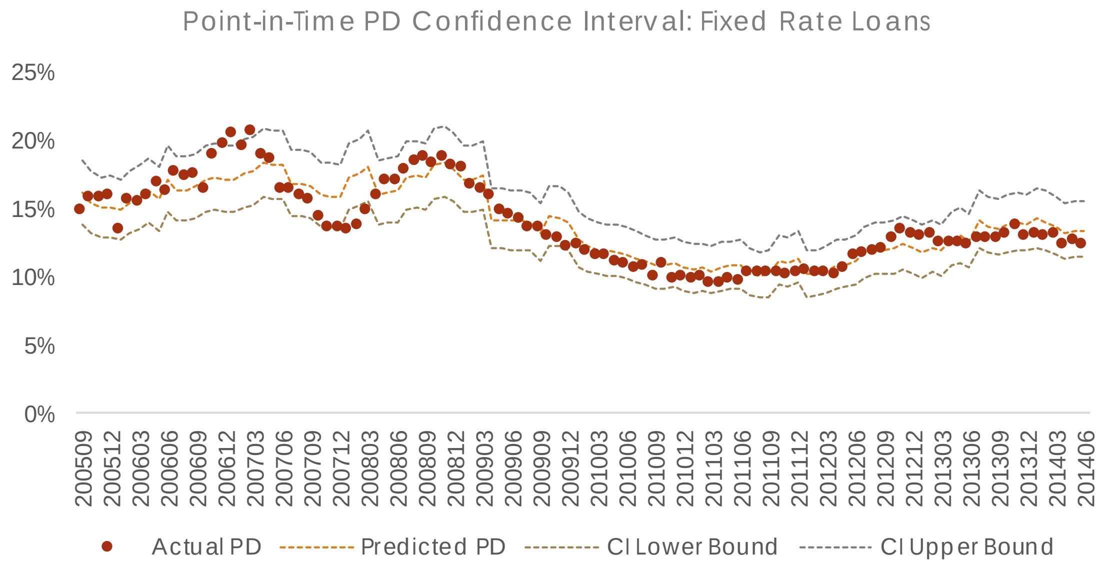


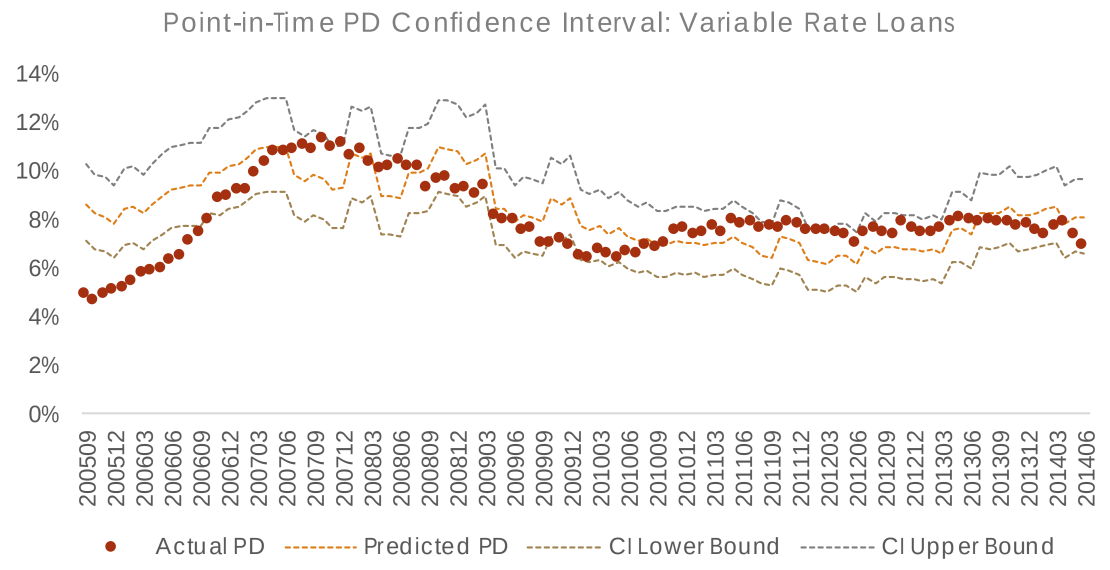
General Asset Correlation Coefficient Formula

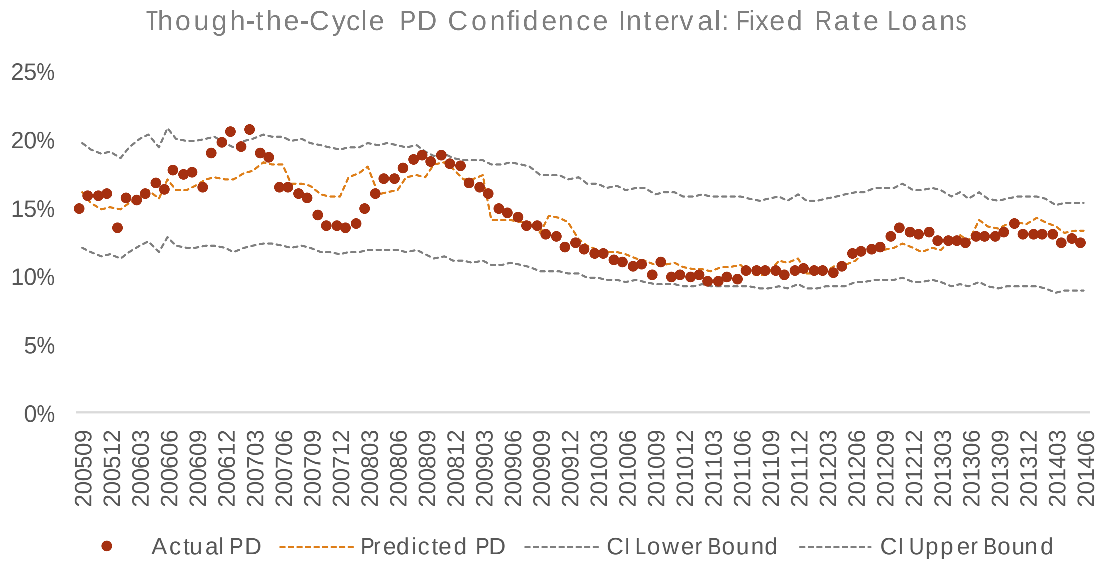
$$\rho = \left[\frac{unexplained\ volatility}{total\ volatility}\right] = \left[\frac{\sigma_E^2(1-r^2)}{1+\sigma_E^2(1-r^2)}\right] = \left[\frac{\beta\sigma_S^2(1-r^2)}{1+\beta\sigma_S^2(1-r^2)}\right]$$

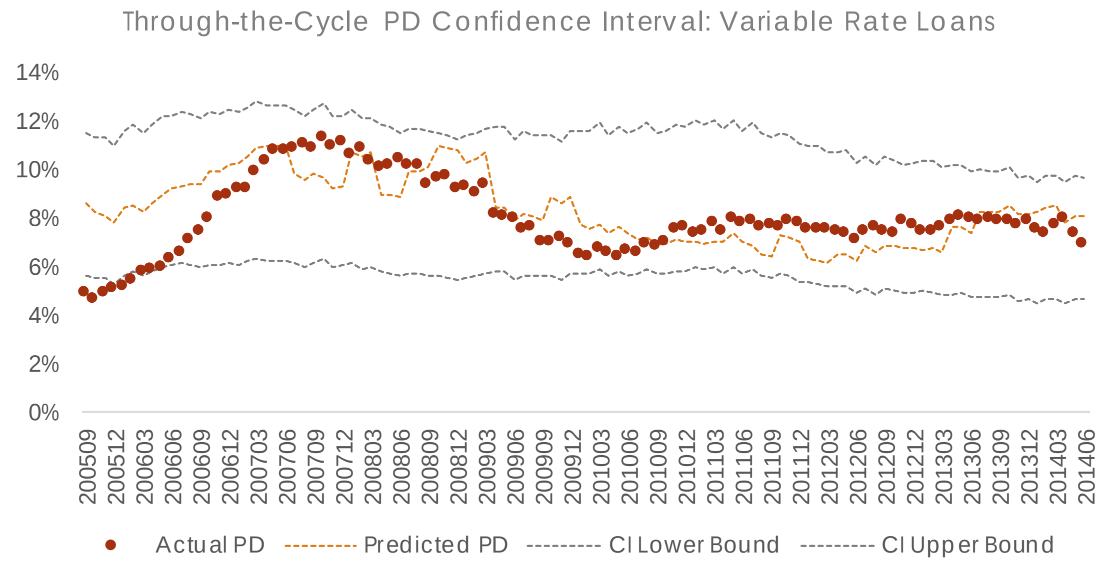
- The asset correlation coefficient is influenced by:
  - Level of risk within the system/economy σ_S^2
  - Pro-cyclicality of portfolio β
  - Level of portfolio's systemic risk that is explained by the model r^2
- To get TTC confidence intervals, simply set $r^s = 0$
- The formula supersedes the generic formulae offered under Basel.

Illustration of Value-at-Risk



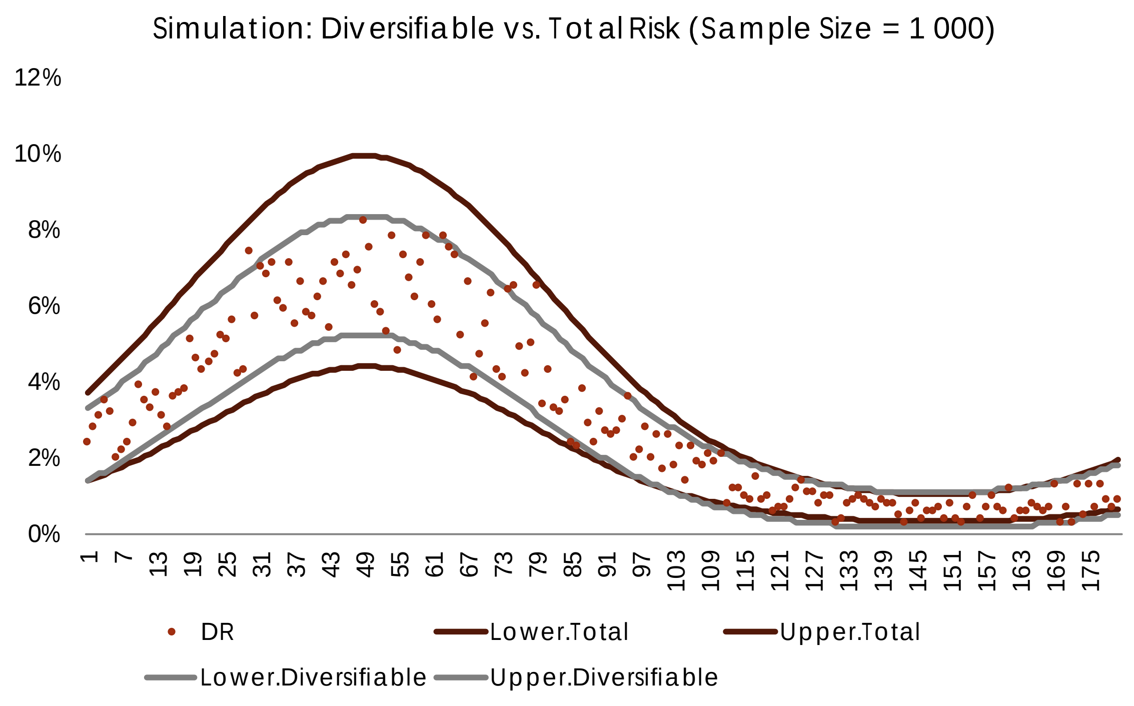


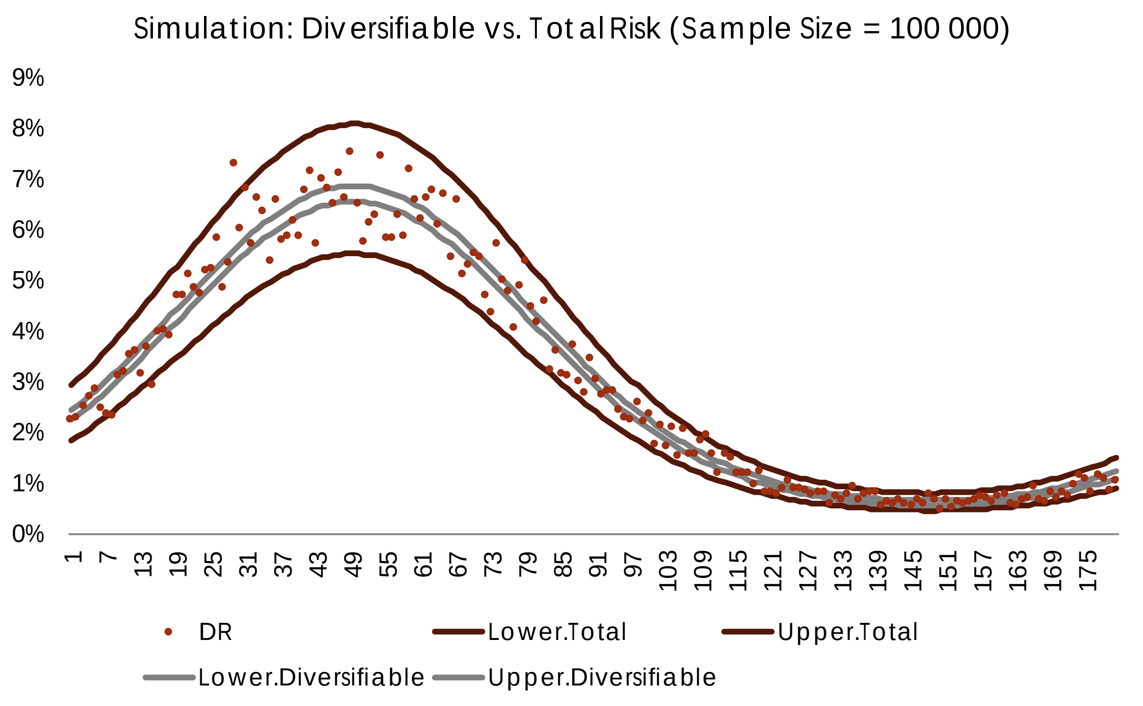




Understanding (Non-)Diversifiable Risk

- For homogenous population, diversifiable risk follows a scaled binomial distribution.
- Due to the asymptotic aspect of LHP assumption, the loss distribution ignores sampling error (diversifiable risk).
- However, observed PD subject to sampling error, which filters through to exogenous component.
- Therefore, the estimate asset correlation coefficient captures total risk.





Cross-Portfolio Aggregation

A formula for cross-portfolio aggregation

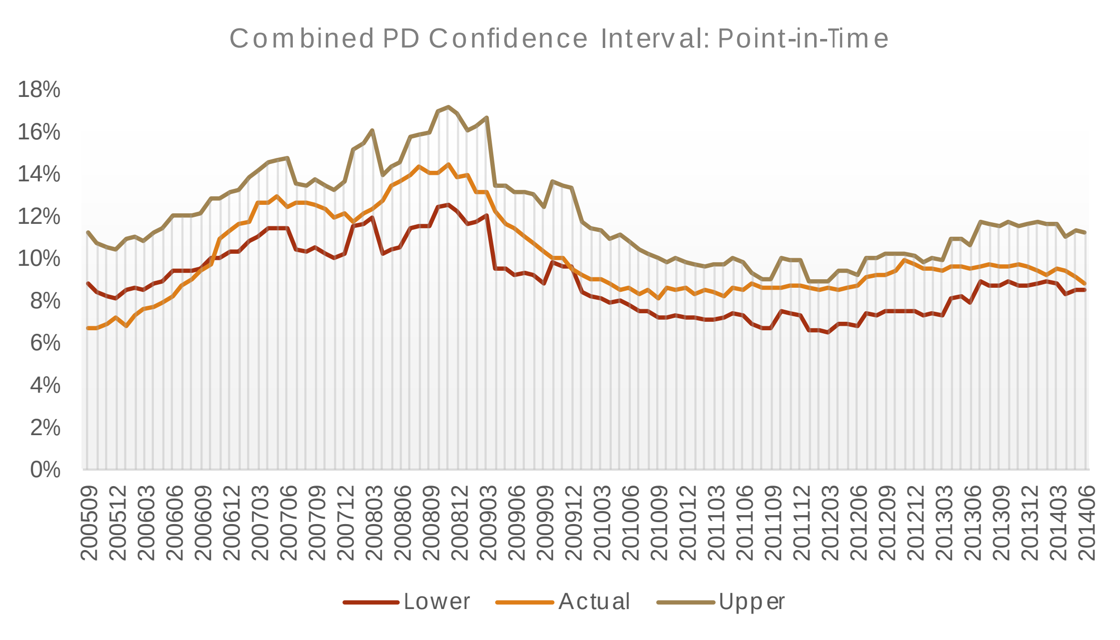
Other uses of the formula

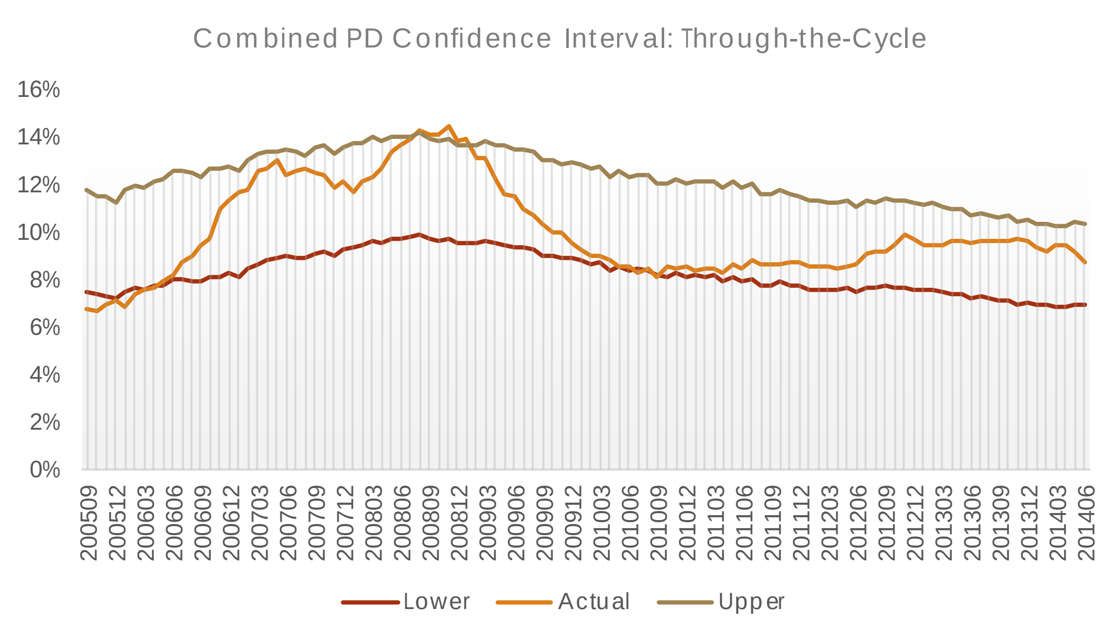
Illustration: South African Portfolio

Formula for Cross-Portfolio Aggregation

- **Aim**: to allow for diversification benefits across portfolios
- **Assumptions**:
  - Assume exogenous error size is small…
  - So that Taylor approximation applies: $p_k(e_{k,t}) \approx \Phi(R_{k,t}) + \phi(R_{k,t})\sigma_{k,e}e_{k,t}$
  - Assume that portfolio errors normally distributed $e_{k,t}$ and linearly correlate
  - Joint loss distribution becomes multivariate normal
- **Practical uses**:
  - Working out required capital for an entire bank
  - Capital budgeting: Determining how much a portfolio contributes to the total risk
  - System analysis: Determining how much a bank contributes to systemic risk

Illustration

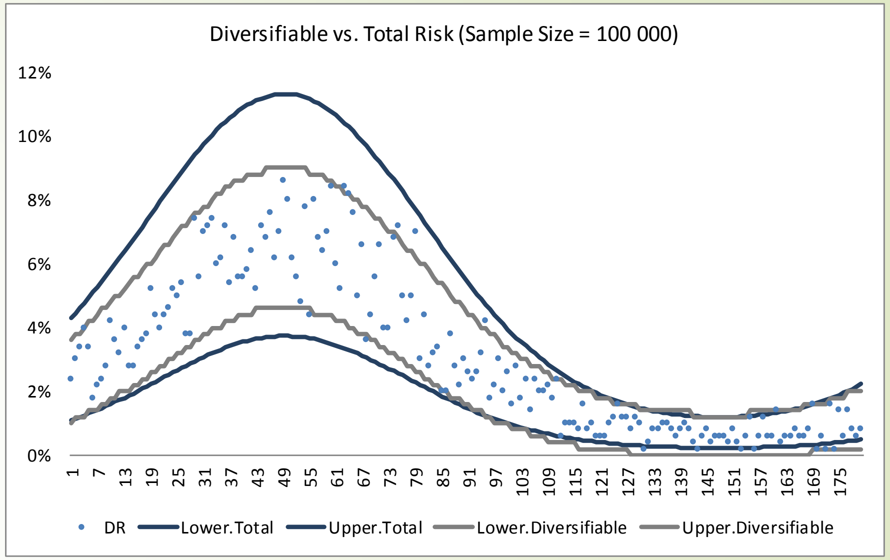


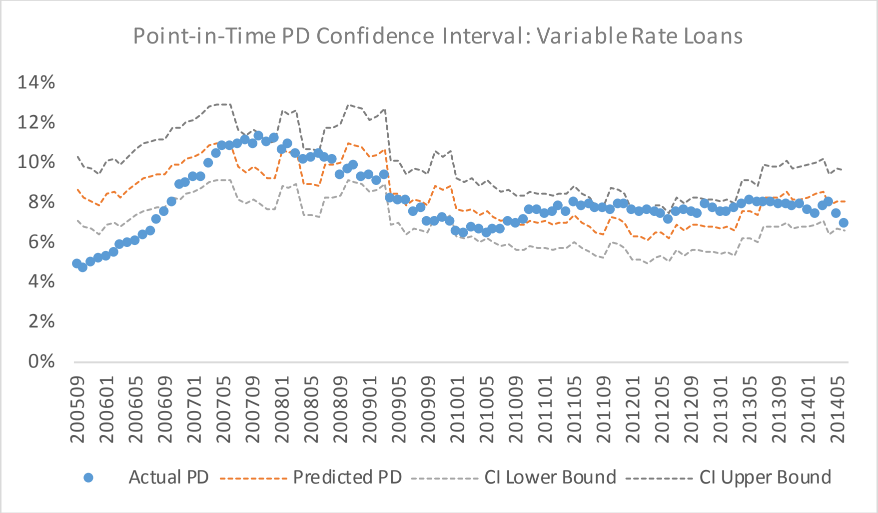


Final Comments on Error Distribution

- Most talk about error distribution looks at the fatness of the tails (Kurtosis)
  - i.e., attempt to capture extreme events.

- This presentation suggests that:
  - For poor-fitting macroeconomic models, autocorrelation matters.
  - For small portfolios with low default rates, skewness matters.
  - For aggregating across portfolio, the joint error distribution matters.





Conclusion

- **Extended EMV Model**
  - Solving (bypassing) the identifiability problem
  - Survival analysis decomposition
- **Unification**
  - Application in scoring, impairment, stress testing, capital management
- **Systemic Risk**
  - Understanding loss aggregation and asset correlation
  - General formula for asset correlation
- **Cross-Portfolio Aggregation**
  - Aggregation across portfolio
- **Further Research**
  - Error distribution, and joint error distribution
  - Combining model with other risk types (e.g., liquidity, operational)

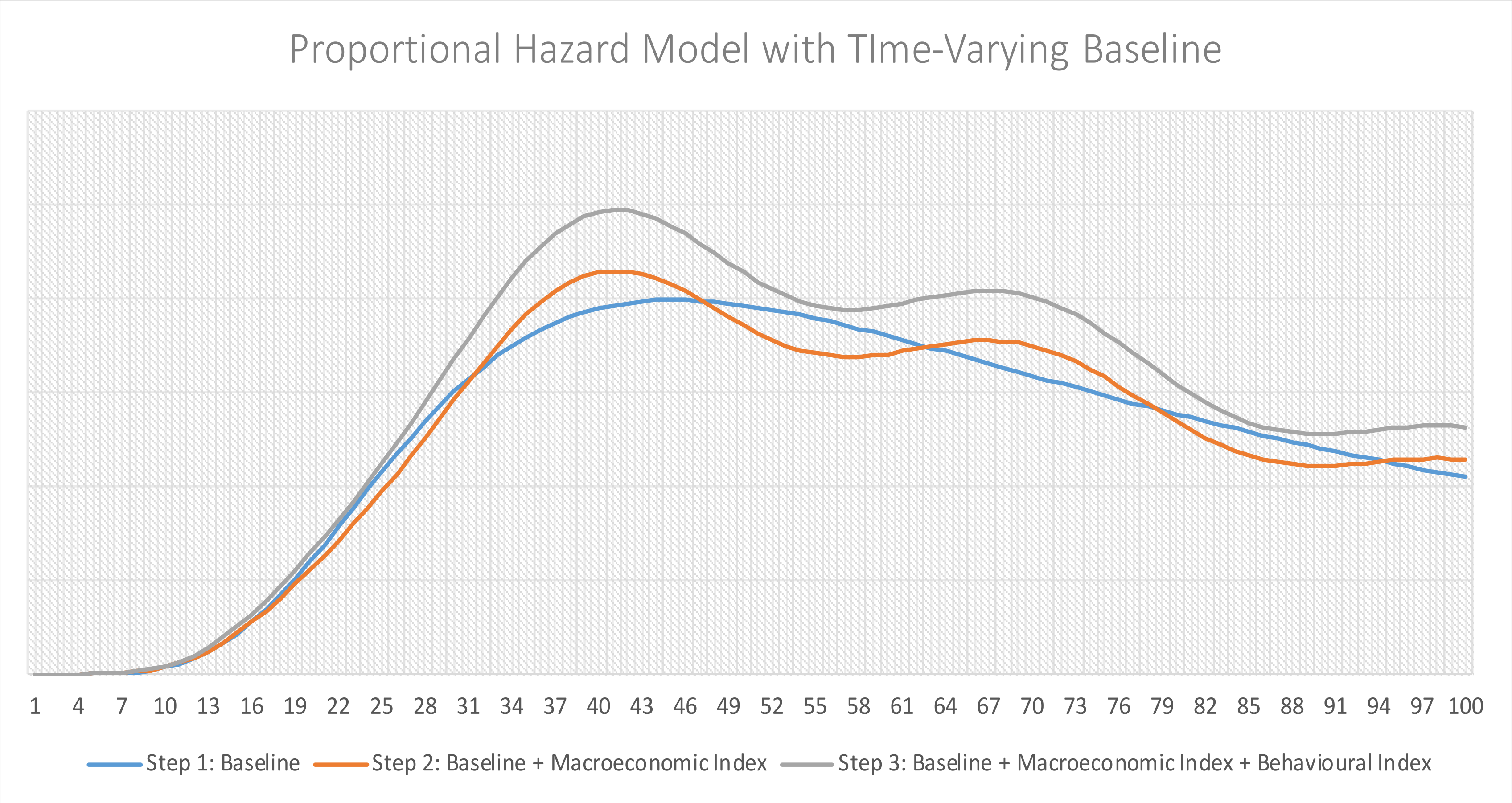
Appendix

Application of EMV-type decomposition to Survival Analysis

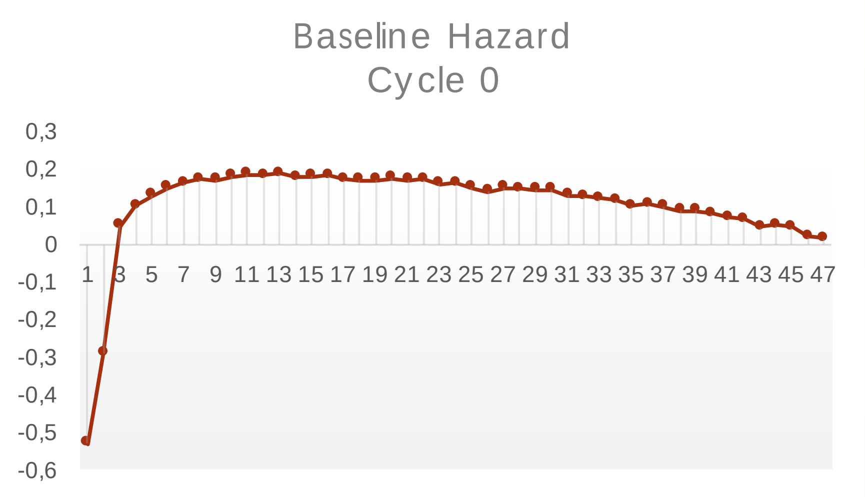
Extension: Survival Analysis Decomposition

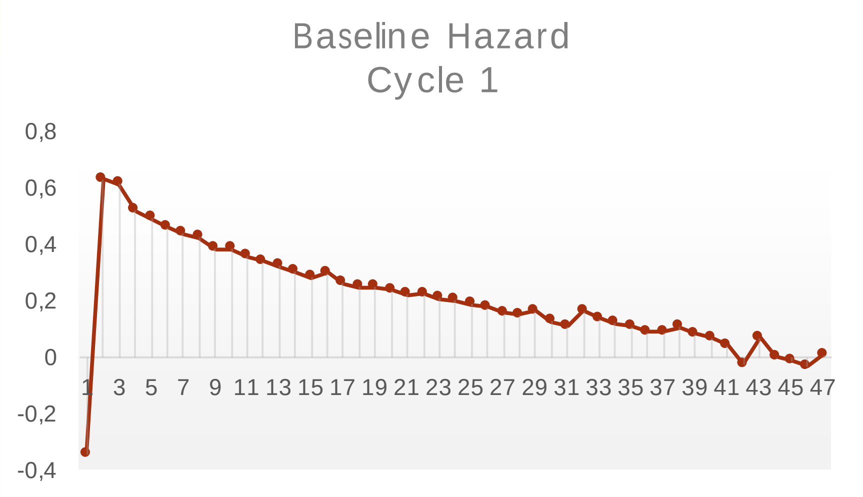
$$h_{j,s}(t) = \Phi\left(\alpha_1 b_t + \alpha_2 \varphi_{G_{j,s}} + \alpha_3 e_s\right) \; or \; h_{j,s}(t) = \Phi(\alpha_1 b_t + \alpha_3 e_s) e^{\varphi_{G_{j,s}}} \equiv b_t e^{\varphi_{G_{j,s}}}$$

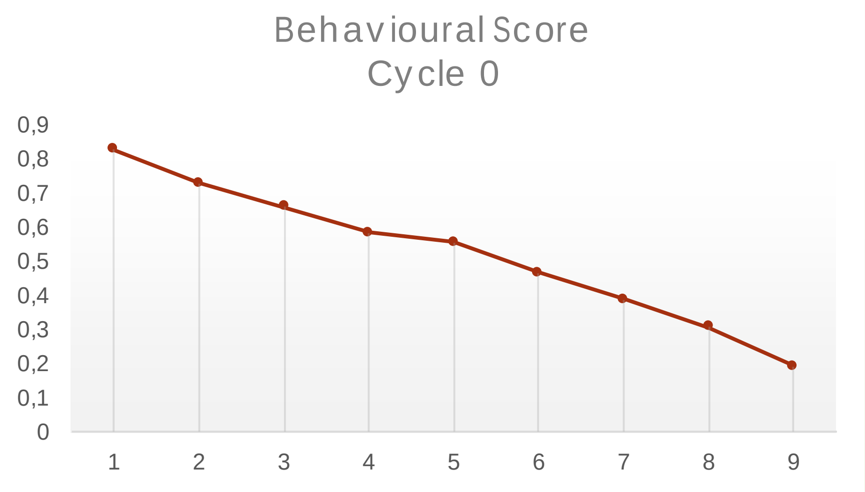
- **Objective**:
- Survival analysis with time varying covariates
- Non-parametric estimation without need for partial-likelihood function

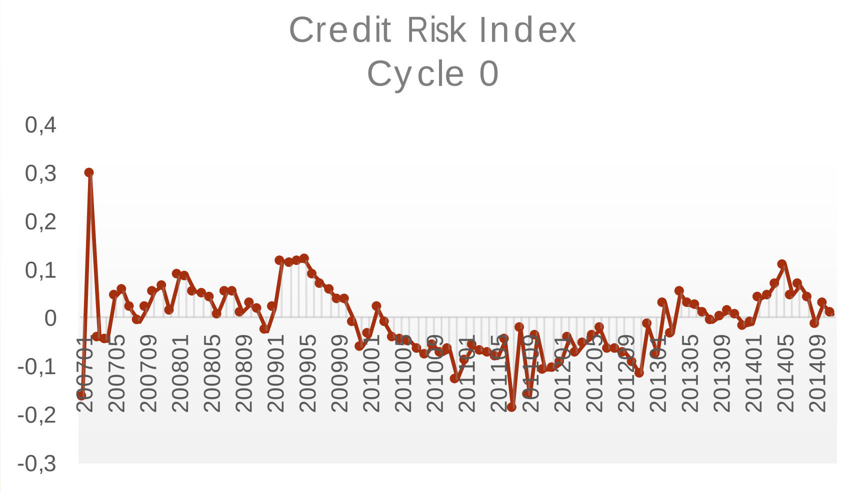


Extension: Survival Analysis Decomposition

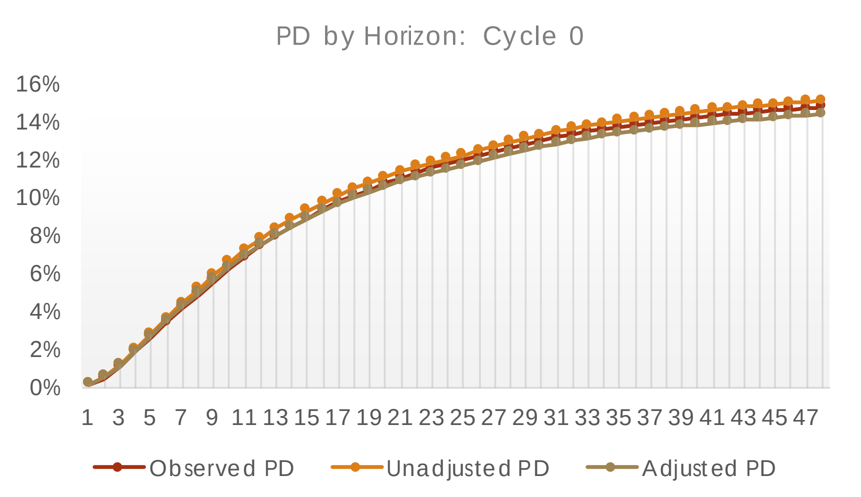


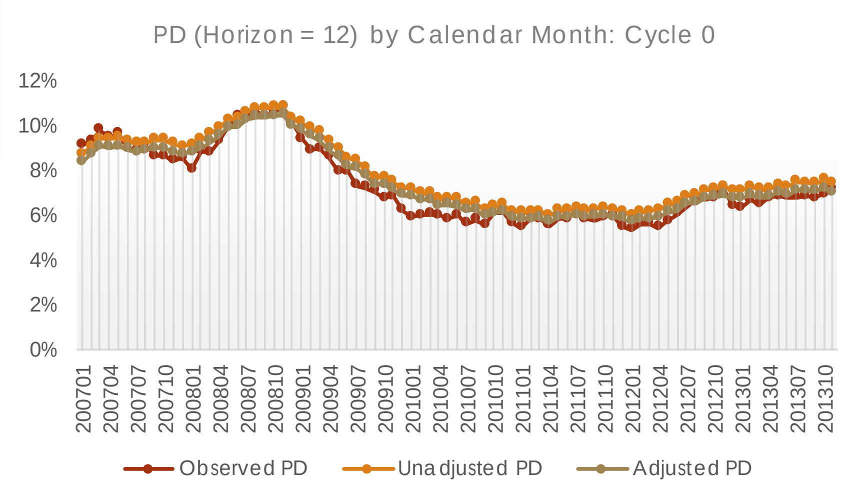


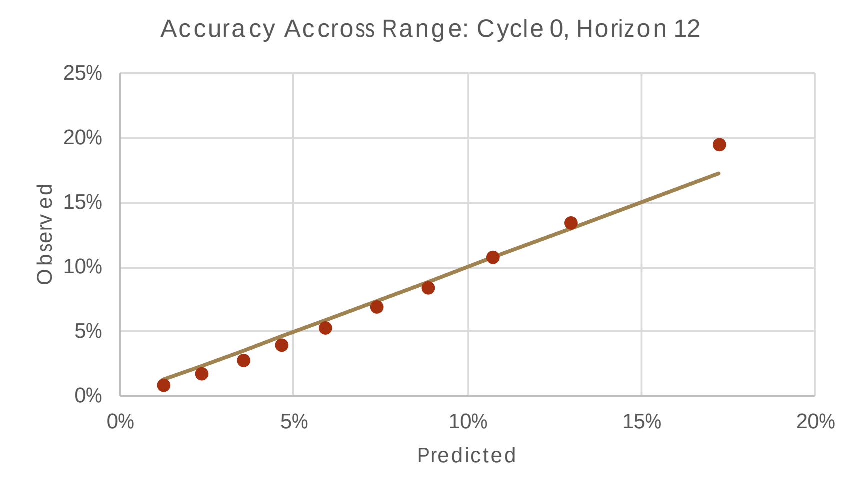


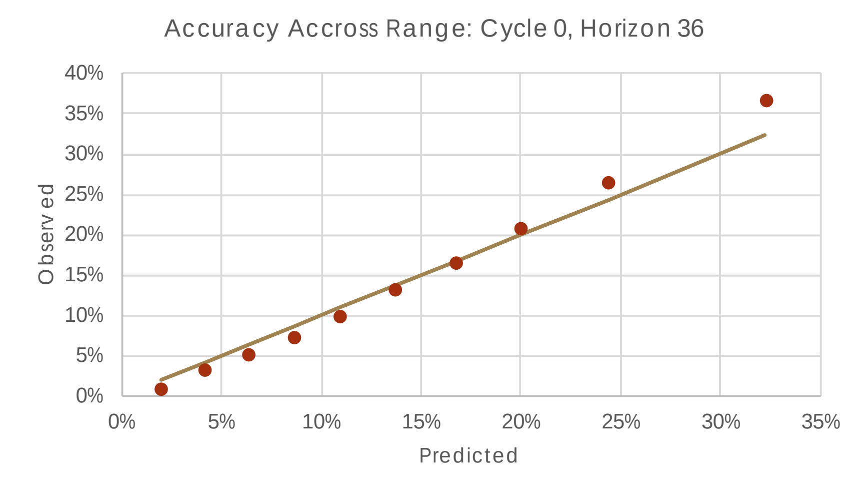


Extension: Survival Analysis Decomposition









EMV Survival Analysis Data Setup

Original Record ID	s	j	$T_{j,s}$	$D_{j,s}$	$G_{j,s}$
1	Jan-10	1	7	0	5
2	Jan-10	2	2	1	6
3	Jan-10	3	3	1	5
4	Feb-10	1	6	0	5
5	Feb-10	2	1	1	8
6	Feb-10	3	2	1	6
7	Mar-10	1	5	0	4
8	Mar-10	3	1	1	7

New Record ID	Original Record ID	s	u	j	t	$d_{j,s,t}$	$G_{j,s}$
1	1	Jan-10	Feb-10	1	1	0	5
2	1	Jan-10	Mar-10	1	2	0	5
3	1	Jan-10	Apr-10	1	3	0	5
4	1	Jan-10	May-10	1	4	0	5
5	1	Jan-10	Jun-10	1	5	0	5
6	1	Jan-10	Jul-10	1	6	0	5
7	1	Jan-10	Aug-10	1	7	0	5
8	2	Jan-10	Feb-10	2	1	1	6
9	2	Jan-10	Mar-10	2	2	1	6
10	3	Jan-10	Apr-10	3	3	1	5
11	3	Jan-10	Mar-10	3	2	1	5
12	3	Jan-10	Feb-10	3	1	1	5
13	4	Feb-10	Mar-10	1	1	0	5
14	4	Feb-10	Apr-10	1	2	0	5
15	4	Feb-10	May-10	1	3	0	5
16	4	Feb-10	Jun-10	1	4	0	5
17	4	Feb-10	Jul-10	1	5	0	5
18	4	Feb-10	Aug-10	1	6	0	5
19	5	Feb-10	Mar-10	2	1	1	8
20	6	Feb-10	Mar-10	3	1	1	6
21	6	Feb-10	Apr-10	3	2	1	6
22	7	Mar-10	Apr-10	1	1	0	4
23	7	Mar-10	May-10	1	2	0	4
24	7	Mar-10	Jun-10	1	3	0	4
25	7	Mar-10	Jul-10	1	4	0	4
26	7	Mar-10	Aug-10	1	5	0	4
27	8	Mar-10	Apr-10	3	1	1	7

Questions

Please contact me at: malwandla@live.co.za