„Rating Properties and their Implications for Basel II – Capital“

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Agenda

- Validation of default probabilities
- Properties of Default Probabilities
- Impact on Basel II-Capital
- Inferential Statistic: “Spiegelhalter test”
- Empirical analysis
- Discussion
Validation of Default Probabilities

- Basel Committee on Banking Supervision proposes new framework for banking regulatory capital based on the inherent credit risk, which depends on forecasted parameters such as
  - Probability of default,
  - Exposure at default,
  - Loss given default and
  - Maturity.

- Challenge: Validation of the forecasted default probabilities ($\hat{p}_i$) by comparing the forecasts with the observed outcomes ($y_i$):
  - Default ($y_i = 1$)
  - No default ($y_i = 0$)
**Validation of Default Probabilities**

- Accuracy (also known as “Calibration-in-the-Small”) is defined as:
  \[ \hat{\pi}_i = \pi_i \] for all \( i \).
- Accuracy is measured by the Mean Square Error (MSE):
  \[ MSE = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{\pi}_i)^2 \]

- The smaller the MSE, the higher the forecasts’ accuracy.
- Weighted penalty function, in which discrepancies between observations and forecasts are weighted by their severity through the quadratic function.
- MSE is the basic element of the further considerations
Properties of Default Probabilities

- By decomposing the MSE, different properties of default probabilities can be shown.

- Decomposition I of MSE:

\[
MSE = \left( \bar{y} - \hat{\pi} \right)^2 + \frac{s_y^2}{\hat{\pi}} + \frac{s_{\hat{\pi}}^2}{\pi} - 2 \cdot s_y \cdot s_{\hat{\pi}} \cdot r_y \hat{\pi}
\]

- Decomposition II of MSE:

\[
MSE = s_{\hat{\pi}}^2 + \sum_y \frac{N_y}{N} (\hat{\pi}_y - \bar{y})^2 - \sum_y \frac{N_y}{N} (\hat{\pi}_y - \hat{\pi})^2
\]
Properties of Default Probabilities

- The following properties of default probabilities can be derived from the accuracy measure MSE:
  - Over-All-Calibration,
  - Uncertainty,
  - Refinement,
  - Association,
  - Discrimination I and
  - Discrimination II.
Impact on Basel II - Capital

- Note: a higher refinement leads to a lower average capital requirement.
Inferential Statistics

- Up to now: descriptive statistics for definition of rating properties
- Defaults or non-defaults are realisations of a random variable:
  \( y_i \) is Bernoulli distributed with parameter \( \pi_i \)
- Thus, MSE is a random variable itself
- The best a forecaster can do is to predict each default probability correct:
  \( H_0 : \hat{\pi}_i = \pi_i \) for all \( i \) : tests on “to be calibrated-in-the-small”

- Then: 
  \[
  E(MSE_{\hat{\pi}_i=\pi_i}) = \frac{1}{N} \sum_{i=1}^{N} \pi_i(1 - \pi_i)
  \]

- Test statistic: 
  \[
  Z_S = \frac{MSE - E(MSE_{\hat{\pi}_i=\pi_i})}{\sqrt{Var(MSE_{\hat{\pi}_i=\pi_i})}} \quad \text{approx.} \sim N(0;1)
  \]

- See Spiegelhalter (1988)
Empirical Analysis

- Comparison of three probit models for the default probabilities. The Models differ in the number of significant risk factors and therefore included information:
  - Model 1:
    - Working capital to total assets.
  - Model 2:
    - Working capital to total assets and
    - Long-term debt to total assets.
  - Model 3:
    - Working capital to total assets (WCA),
    - Long-term debt to total assets (LDA) and
    - Change in US real gross domestic product (GDP).
Empirical Analysis

- Data set: typical small-sized corporate loan portfolio from 1997 to 2003
- 18,246 observations and 228 defaults
- Estimation Sample: 1997 to 2002
- Validation Sample: 2003 with 2,223 observations and 36 defaults
- Parameter estimates:

<table>
<thead>
<tr>
<th>Model</th>
<th>Variable</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>Standardised Estimates</th>
<th>p-Value</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Intercept</td>
<td>-2.1808</td>
<td>0.0295</td>
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<td>3</td>
<td>Intercept</td>
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<tr>
<td></td>
<td>GDP</td>
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<td>0.0173</td>
<td>-0.1187</td>
<td>&lt;.0001</td>
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<tr>
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<td>LDA</td>
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<td>-0.0885</td>
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</table>
Empirical Analysis

- Characteristics of the forecasts:

<table>
<thead>
<tr>
<th>Model</th>
<th>$\bar{y}$</th>
<th>$\bar{\pi}$</th>
<th>$\bar{\pi}_{y=0}$</th>
<th>$\bar{\pi}_{y=1}$</th>
<th>$s_{\bar{\pi}}$</th>
<th>Average Capital Requirement</th>
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<tbody>
<tr>
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<td>0.00840</td>
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**Empirical Analysis**

- Decomposition I of MSE

<table>
<thead>
<tr>
<th>Model</th>
<th>Accuracy</th>
<th>Over-All-Calibration</th>
<th>Variance of Forecast Error</th>
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<td>2</td>
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Empirical Analysis

- Decomposition II of MSE

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<th>Model</th>
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<th>Refinement</th>
<th>Discrimination I</th>
<th>Discrimination II</th>
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<td>0.0157849</td>
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</table>
Empirical Analysis

- Spiegelhalter test: \( H_0 : \hat{\pi}_i = \pi_i \) for all \( i \)

<table>
<thead>
<tr>
<th>Model</th>
<th>MSE</th>
<th>p-Value</th>
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<tr>
<td>3</td>
<td>0.0158550</td>
<td>0.3591</td>
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</tbody>
</table>

- The null hypothesis can be rejected for model 1 (alpha = 0.05)
- Model 2 and 3 are assumed “to be calibrated-in-the-small”
Calibration-in-the-Small can be achieved (among others) by different dimensions:
- Refinement,
- Discrimination and
- Over-All-Calibration.

Importance of these properties depends especially on the type of portfolio.

Measures are random like the default events themselves, i.e., inferential statistics based on the tests of hypotheses should be conducted.
Discussion

- Note: Validation is not solely a statistical exercise. All of the approaches including ours are especially limited with regard to assumptions and data.

- A broader validation approach should encompass due diligence of the
  - Model and its alternatives,
  - Development processes,
  - Conceptual soundness and
  - Previous statistical studies.
Thank you for your attention

Please
- Feel free to share comments or ask any questions you may have.
- Let me know if you are interested in the ongoing research.

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