Optimizing credit limit policies to maximize customer lifetime value

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Outline

- Motivation
- Markov decision process (MDP)
- The use of the behavioral score
- Low default account
- A case study
- Future research
Credit card market: key figures

**Total value of transactions (Purchases in the UK)**

- **Credit card market**:
  - Prominent payment tool
  - High penetration
  - Key payment method in e-shopping
  - Widely used by retailers

*Source: APACS, the UK payments association*

- **In 2005...**
  - Number of credit card holders reached 31.6 million
  - The average number of cards per person was 2.4 credit cards

*In 2005...*
Challenges

- Usage rate slowdown
- 59% of customers are transactors
- Intense competition among credit card issuers

Question

*How can banks raise the credit card portfolio by encouraging customers to increase spending?*
Possible policies

- Offer additional credit products
- Offer 0% interest rate on balance transfer
- Extend 0% interest rate on new purchases
- Reduce pricing (adjust APR, Annual Percentage Rate)
- Increase credit limit

*Manage the credit limit by Markov decision process
[Bank One Card Services, 2003]*

Bank One Services’ Approach

Our Approach
The choice of behavioural score

- Default indicator
- Popular
- Properly capture repayment behavioral
- Reduce computational time
- Link credit limit and behavioural score dynamically

Markov Decision Process

The formulation of MDP requires:

- A set of states (classification of customers)
- Transition probabilities (from one state to another)
- Revenue corresponding to different states
- Possible actions

The deliverables would be

- a set of optimal actions for each state
Choice of MDP

Advantage
- A lot of time-series credit card records
- Possible to estimate the expected profit
- Possible to apply different actions

Hurdle
- Markovian assumption
  - assume a state depends mainly on information concerning the current position of the account and its recent history, which means that the probability of moving from state $x$ to state $y$ depends only on $x$, conditionally independent of which states and actions preceded the move to state $x$

State Variables

- Time $t$ : in month
- State variables:
  - Credit limits, $l$
  - Behavioural score, $i$
Model

- Transition probability:
  \[ P( i' \mid l, i ) = \text{the probability that the system moves from credit limit } l \text{ and behaviour score } i \text{ to a behaviour score of } i' \text{ at one time period} \]

- Optimality equation:
  \[
  V_t(l, i) = \text{Max}_{l' \geq l} \left\{ \sum_{i'} p(i' \mid l, i) [r(i' \mid l, i) + \lambda V_{t-1}(l', i')] \right\}
  
  \text{the discounted profit (with discount factor } \lambda \text{) over the next } t \text{ month given the customer with credit limit } l \text{ and behavioural score } i
  
Profit Function

Balance at time t: \( B_t = B_{t-1} + N_{t-1} - P_{t-1} \)
Profit in period t: \[
\text{Profit in period } t = N_{t-1} f + (B_{t-1} - P_{t-1}) r
= N_{t-1} f + (B_{t-1} - B_{t-1} + B_t - N_{t-1}) r
= N_{t-1} (f - r) + B_t r
\]

If we assume \( r = f \text{ and LGD equals to } 1 \) (where D represents default accounts)

\[
\text{Estimate profit at } t = EP_t = \begin{cases} 
   rB_t, & \forall t \neq D \\
   -B_t, & \text{otherwise}
\end{cases}
\]

Validation:

i.e. assume \( AP_t = \text{the actual profit}, \ Diff = AP_t - EP_t \)

Test:

\( \text{Diff follows normal distribution with mean equals to zero} \)
Low default portfolios

- High behavioral score accounts
- Seldom or “No” default case in the sampling period
- Build a “structural zeros” model
- Underestimate the lost
- Generate unusual and not practical policy

Example (LDP)

Definition:
1 – Excellent
2 – Good
3 – Bad

\( n_{ij} \) – number of accounts transit from state i to j
\( a_{ij} \) – transition probability from state i to j

\[
\begin{align*}
1_{13} &= 1 \\
a_{13} &= 0.0002 \\
a_{33} &= 1 \\
n_{23} &= 8 \\
a_{23} &= 0.0027
\end{align*}
\]

Credit limit : £5,000 +
Total cases : 10,000
5,500 in Excellent
3,000 in Good

Credit limit : £2,000 to £3,000
Total cases : 500
200 in Excellent
300 in Good

No default
Low default portfolios (LDP)

- Define:
  1. PD for the accounts with best behavioral score: \( p(D \mid l, I) \)
  2. Accounts with states \( (l, i), (l, i+1), \ldots, (l, I) \) are the low default portfolios
  3. \( N(l, i) \) = the total number of accounts in \( (l, i), (l, i+1), \ldots, (l, I) \)
  4. \( N_D(l, i) \) = the total number of accounts in \( (l, i), (l, i+1), \ldots, (l, I) \) transit to default

- Apparently:
  \[ p(D \mid l, i) \geq p(D \mid l, i+1) \geq \ldots \geq p(D \mid l, I) \]

- If we assume:
  1. \( p(D \mid l, i) = p(D \mid l, i+1) = \ldots = p(D \mid l, I) \)
  2. Default incidence follows the binomial distribution

Then with a confidence level \( 1-\gamma \), there are \( N_D(l, i) \) default in the low default portfolio:

\[
1 - \gamma \leq \sum_{j=0}^{N_D(l, i)} \binom{N(l, i)}{j} p(D \mid l, I)^j (1 - p(D \mid l, I))^{N(l, i) - j}
\]

Case Study

- Credit card dataset from a UK bank
- From 2001 to 2004 inclusive
- Monthly records
- Attributes: credit limit, behaviour score, special account status, account balance
- 50,000+ cases for analysis
- APR: 24%
- Discount value: 0.995
### Classification of states

<table>
<thead>
<tr>
<th>#</th>
<th>Behaviour Score/Account Description</th>
<th>#</th>
<th>Credit limit (in £)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Closed</td>
<td>0</td>
<td>missing/closed</td>
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<td>Inactive</td>
<td>1</td>
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<tr>
<td>2</td>
<td>Bad (Bankruptcy or charge-off)</td>
<td>2</td>
<td>501-1000</td>
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<tr>
<td>3</td>
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<td>4</td>
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<td>6</td>
<td>Score 3</td>
<td>6</td>
<td>3501-4500</td>
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<tr>
<td>7</td>
<td>Score 4 (highest score group)</td>
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<td>4501-5500</td>
</tr>
<tr>
<td></td>
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<td>8</td>
<td>≥ 5501</td>
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### Transition matrix

#### Credit Limit: ≤ £500 (# of transition 148,207)

<table>
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<tr>
<th>Behaviour Score</th>
<th>Closed</th>
<th>Inactive</th>
<th>Bad (Bankruptcy or charge-off)</th>
<th>In risk</th>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
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<tbody>
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<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
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<td>0.000000</td>
<td>0.003170</td>
<td>0.002662</td>
<td>0.000317</td>
<td>0.00016</td>
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<tr>
<td>Bad</td>
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<td>0.000000</td>
<td>0.000000</td>
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<tr>
<td>In risk</td>
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<td>0.000000</td>
<td>0.247706</td>
<td>0.293578</td>
<td>0.394495</td>
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<tr>
<td>Score 1</td>
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#### Credit Limit: ≥ £501 (# of transition 430,082)

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<th>Bad (Bankruptcy or charge-off)</th>
<th>In risk</th>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
</tr>
</thead>
<tbody>
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<td>0.123014</td>
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Transition matrix
Credit Limit: ≤ £500 (# of transition 148,207)

<table>
<thead>
<tr>
<th>Beh.Score</th>
<th>Closed</th>
<th>Inactive</th>
<th>Bad</th>
<th>In risk</th>
<th>Score 1</th>
<th>Score 2</th>
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<tr>
<td>In risk</td>
<td>0.064220</td>
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Credit Limit: ≥ £5501 (# of transition 430,082)

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Profit

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<th>Monthly reward in £</th>
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<th>In risk</th>
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✿ With the same credit limit, consumers with lower behaviour score generates higher reward
### Results

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<tr>
<th>Credit Limit</th>
<th>Optimal Policy</th>
<th>Behaviour Score</th>
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<tr>
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</tbody>
</table>

Index:
- C : Closed
- 1 : £500
- 2 : £501-1000
- 3 : £1001-1500
- 4 : £1501-2500
- 5 : £2501-3500
- 6 : £3501-4500
- 7 : £4501-5500
- 8 : ≥£5501

### Conclusion

- To link the credit limit policy to customers’ behavioural score
- A set of optimal policy with expected profit estimation
- Fast generation time
Future research

- Classification – decision tree model
- Segmentation – mover-stayer model
- Economic variables – logistics regression model

Q&A

Thank you for your attention!