Monitoring Credit Portfolios using Survival Analysis

Axel Gandy

Department of Mathematics
Imperial College London
http://www.ma.ic.ac.uk/~agandy

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Setup

- Customers take out a loan at known times
- Repay for some time until
  - they finished repaying or
  - a default occurs.
- Lexis Diagramm:
Detecting Changes in the Defaults in a Credit Portfolio

- **Main steps:**
  - Construct model for in-control behaviour
  - Set up monitoring scheme
  - If alarm: investigate

- **Use Survival Analysis** (How to? Why?)

- Not restricted to defaults in credit portfolios: applies to other type of events (e.g. churn, purchase, fraud)
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How do changes affect default rates? Simulation Example

- 5 year loans; early repayment possible
- Default distribution:
  Low constant hazard rate during first year,
  higher constant hazard rate after first year
- Arrival scenarios:
  - Steady (Poisson Process)
  - Doubling of new customers (at time 1)
  - Halving of new customers (at time 1)
- Scenarios for change in default rates:
  - No change
  - Crisis (at $t = 1.5$) - all customers affected
  - Bad new customers (at $t = 1.5$) - only new customers affected
Yearly Default Rates (6 month gliding window)

- steady
- no change
- crisis
- bad new customers

Yearly default rate (based on past 6 months)

$t$ [years]

0 1 2 3 4 5 6

0.010 0.012 0.014 0.016 0.018 0.020

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Monitoring Credit Portfolios using Survival Analysis
Yearly Default Rates (6 month gliding window)

- **steady**
- no change
- crisis
- bad new customers

- **doubling**
- no change
- crisis
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<table>
<thead>
<tr>
<th>t [years]</th>
<th>Yearly Default Rate (based on past 6 months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.010</td>
</tr>
<tr>
<td>1</td>
<td>0.012</td>
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<tr>
<td>2</td>
<td>0.014</td>
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<tr>
<td>3</td>
<td>0.016</td>
</tr>
<tr>
<td>4</td>
<td>0.018</td>
</tr>
<tr>
<td>5</td>
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Monitoring Credit Portfolios using Survival Analysis
Yearly Default Rates (6 month gliding window)

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- **bad new customers**

- **doubling**
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- **halving**
- **no change**
- **crisis**
- **bad new customers**

**t** [years]

yearly default rate (based on past 6 months)
CUSUM charts (Page, 1954)

- Default rates $X_1, X_2, \ldots$
- $R_t = \sum_{i \leq t} (X_i - k)$
- $S_t = R_t - \min_{s \leq t} R_s$
- Signal at $\tau = \min \{ t : S_t \geq c \}$
CUSUM charts (based on half-monthly default rates)

Threshold such that $P(\text{false alarm in 10 years}) \leq 0.01$
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Monitoring Default Rates - Lexis Diagramm

![Lexis Diagramm](image-url)

- **Calendar Time (years)**
- **Time at Risk (years)**

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Monitoring Credit Portfolios using Survival Analysis
Fixed Follow-Up

- Model for probability of default up to a given time $t_0$

$$P(T_i \leq t_0 | Z_i)$$

(many choices, e.g. logistic regression)

- CUSUM chart based on likelihood ratio between an alternative and the in-control model
Fixed Follow-Up - Lexis Diagramm

\( t_0 = 2 \text{ years, individuals considered at time } B_i + t_0 \)
Fixed Follow-Up

$t_0 = 2$ years, individuals considered at time $B_i + t_0$
Fixed Follow-Up - Use Information Immediately?

- Not a good idea! → population effects!
- Example:
  - follow-up time $t_0 = 2$
  - arrival of new customers changes at $t = 1$
  - use default information about individuals at time $B_i + \min(X_i, t_0)$

- If no early repayment, i.e. no censoring:
  - # of defaults changes immediately
  - # of non-defaults changes only at $t = 3$
- With early repayment:
  - more complicated effects
Survival Analysis in Credit Scoring

- A lot of models to choose from, e.g. Cox’s proportional hazard model.
- Use in credit risk, see e.g. Banasik et al. (1999), Stepanova & Thomas (2002, 2001)
- Early repayment (censoring) is dealt with automatically
Detecting a Proportional Change in the Hazard

\[ B_1 \leq B_2 \leq \ldots \] calendar times at which individuals arrive

\[ T_i \] survival time

\[ C_i \] censoring time

\[ h_i(s) \] in control hazard rate of \( T_i \)

\[ \rho h_i(s) \] hazard rate after change point

\[ X_i(t) = \min(T_i, C_i, (t - B_i)^+) \] time at risk up to \( t \)

\[ \delta_i(t) = I\{T_i \leq X_i(t)\} \] indicator for observed default up to \( t \)

\[ N(t) = \sum_i \delta_i(t) \] number of defaults up to time \( t \)

\[ \Lambda(t) = \sum_i \int_0^{X_i(s)} h_i(s) ds \] intensity in control up to time \( t \)

\[ \rho \Lambda(t) \] intensity out of control up to time \( t \)

Log-likelihood ratio:

\[ R(t) = \log(\rho) N(t) - (\rho - 1) \Lambda(t) \]

\[ S(t) = R(t) - \min_{0 \leq u \leq t} R(u); \quad \tau = \min\{t : S(t) > c\} \]
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Lexis Diagramm - Survival Analysis

![Lexis Diagramm](image)

- **Calendar time [years]**: 0, 1, 2, 3, 4, 5, 6
- **Time at risk [years]**: 0, 1, 2, 3, 4, 5

This diagram illustrates the relationship between calendar time and time at risk in a survival analysis context.
CUSUM charts (Survival Analysis)

- steady
- no change
- crisis
- bad new customers

- doubling
- no change
- crisis
- bad new customers

- halving
- no change
- crisis
- bad new customers

S(t)

0 1 2 3 4 5 6
0 5 10 15

t [years]

Imperial College London

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### Hitting Times in the Example [months after change]

<table>
<thead>
<tr>
<th></th>
<th>steady</th>
<th></th>
<th>doubling</th>
<th></th>
<th>halving</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>crisis</td>
<td>bnc</td>
<td>crisis</td>
<td>bnc</td>
<td>crisis</td>
</tr>
<tr>
<td>Discrete CUSUM</td>
<td>1.5</td>
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<td>6</td>
<td>27.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Fixed Follow-Up</td>
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<td>29.06</td>
<td>12.65</td>
<td>26.14</td>
<td>12.65</td>
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<tr>
<td>Surv. Anal. CUSUM</td>
<td>1.63</td>
<td>21.61</td>
<td>1.6</td>
<td>19.89</td>
<td>1.56</td>
</tr>
</tbody>
</table>

- Discrete CUSUM = direct monitoring of default rates
Comments

- General methodology for monitoring with Survival Analysis: Gandy et al. (2009)

- Describes e.g. how to set thresholds: Explicit computations possible for
  - $\mathbb{E}(N(\tau))$
  - $P(N(\tau) \leq k)$

- Also possible: Monitoring against a decrease in events
Why use Survival Analysis for Monitoring?

- Quick detection (all available information is used)
- No problem with population effects


