Specialized Lending Rating Model using AHP

Credit Risk Modelling

Edinburgh, 28th August 2013.
Agenda

- Specialized lending
- AHP methodology
- IPRE model
- Validation
- Conclusion

Specialized lending
Specialized lending

- Specialized lending - 4 subclasses:
  - Income Producing Real Estate (IPRE)
  - Project Finance
  - Object Finance
  - Commodities Finance

- Basel Committee approaches:
  - Standardized approach
  - Foundation IRB
  - Advanced IRB
  - Slotting approach (only for specialized lending)
AHP METHODOLOGY

Credit Risk Modelling

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Analytical hierarchy process (methodology overview)

- Introduced by Thomas L. Saaty
- Hierarchical structure – goal on top, criteria and subcriteria in the middle, options on bottom (example of one criteria and one subcriteria level):
Analytical hierarchy process (1st step)

For each subcriteria construct pairwise comparison matrices and corresponding priority vector:

e.g. SC₁:

\[
\begin{pmatrix}
1 & a_{12} & a_{13} \\
1/a_{12} & 1 & a_{23} \\
1/a_{13} & 1/a_{23} & 1
\end{pmatrix}
\]

Normalization calculation of eigen vector & eigen value

\[
\begin{pmatrix}
x_1 \\
y_1 \\
z_1
\end{pmatrix}
\]

\[x_1 + y_1 + z_1 = 1\]
Pairwise comparison

How is pairwise comparison conducted? Using scale (1-9):

<table>
<thead>
<tr>
<th>Intensity of Importance</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
</tr>
<tr>
<td>2</td>
<td>Weak or slight</td>
</tr>
<tr>
<td>3</td>
<td>Moderate importance</td>
</tr>
<tr>
<td>4</td>
<td>Moderate plus</td>
</tr>
<tr>
<td>5</td>
<td>Strong importance</td>
</tr>
<tr>
<td>6</td>
<td>Strong plus</td>
</tr>
<tr>
<td>7</td>
<td>Very strong or demonstrated importance</td>
</tr>
<tr>
<td>8</td>
<td>Very, very strong</td>
</tr>
<tr>
<td>9</td>
<td>Extreme importance</td>
</tr>
</tbody>
</table>

Construct positive reciprocal matrix:

\[
\begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\
    a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\
    \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
    a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & \vdots \\
    \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
    a_{n1} & a_{n2} & \cdots & \cdots & \cdots & a_{nn}
\end{bmatrix}
\]

where:

\[
a_{11} = a_{22} = \ldots = a_{nn} = 1, \\
a_{ji} = a_{ij}^{-1}
\]
Consistency index and consistency ratio

How to check whether the weights in comparison matrix consistent? We calculate eigenvalues of comparison matrices:

- Condition for perfectly consistent weights: $a_{ik} = a_{ij}a_{jk}$

- Condition for perfectly consistent matrices: $\lambda_{\text{max}} = n$

Consistency index: $CI = \frac{\lambda_{\text{max}} - n}{n - 1}$

Consistency ratio: $CR = \frac{CI}{RI}$

- $CR \leq 0.1 \rightarrow$ acceptable inconsistency

Random consistency index (RI) by Saaty:

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>RI</td>
<td>0</td>
<td>0</td>
<td>0.58</td>
<td>0.9</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
</tr>
</tbody>
</table>
Analytical hierarchy process (2nd step)

For each subcriteria group construct matrices out of eigen vectors:

- **3 x 1**
  - \( x_1 \)
  - \( y_1 \)
  - \( z_1 \)
  - \( x_2 \)
  - \( y_2 \)
  - \( z_2 \)

- **3 x 2**
  - \( x_3 \) \( x_4 \)
  - \( y_3 \) \( y_4 \)
  - \( z_3 \) \( z_4 \)

- **3 x 2**
  - \( x_5 \) \( x_6 \)
  - \( y_5 \) \( y_6 \)
  - \( z_5 \) \( z_6 \)
Analytical hierarchy process (3rd step)

For each criterion construct pairwise comparison matrix and corresponding priority vector:

\[
\begin{array}{c|cc}
\text{SC1} & \text{SC2} \\
\hline
\text{SC1} & 1 & b_{12} \\
\text{SC2} & 1/b_{12} & 1 \\
\end{array}
\]

\[
\text{i}_1 + j_1 = 1
\]
Analytical hierarchy process (4th step)

For each criteria calculate priority vector:

\[
\begin{align*}
&\begin{array}{ccc}
3 \times 2 & \times & 2 \times 1 & = & 3 \times 1 \\
& \begin{array}{ccc}
x_1 & x_2 \\
y_1 & y_2 \\
z_1 & z_2 \\
\end{array} & \begin{array}{c}
i_1 \\
j_1 \\
\end{array} & = & \begin{array}{c}
k_1 \\
m_1 \\
n_1 \\
\end{array} \\
\end{array} \\
&\begin{array}{ccc}
3 \times 2 & \times & 2 \times 1 & = & 3 \times 1 \\
& \begin{array}{ccc}
x_3 & x_4 \\
y_3 & y_4 \\
z_3 & z_4 \\
\end{array} & \begin{array}{c}
i_2 \\
j_2 \\
\end{array} & = & \begin{array}{c}
k_2 \\
m_2 \\
n_2 \\
\end{array} \\
\end{array} \\
&\begin{array}{ccc}
3 \times 2 & \times & 2 \times 1 & = & 3 \times 1 \\
& \begin{array}{ccc}
x_5 & x_6 \\
y_5 & y_6 \\
z_5 & z_6 \\
\end{array} & \begin{array}{c}
i_3 \\
j_3 \\
\end{array} & = & \begin{array}{c}
k_3 \\
m_3 \\
n_3 \\
\end{array} \\
\end{array}
\end{align*}
\]
Construct matrix out of eigen vectors and construct pairwise comparison matrix, corresponding priority vector and eigen value:

\[
\begin{array}{c}
\text{GOAL} \\
\text{C1} \\
\text{SC1} \\
\text{OPTION1} \\
\text{C2} \\
\text{SC2} \\
\text{OPTION2} \\
\text{C3} \\
\text{SC3} \\
\text{OPTION3} \\
\end{array}
\]

\[
\begin{array}{c}
\begin{bmatrix}
k_1 \\
m_1 \\
n_1 \\
\end{bmatrix} \\
\begin{bmatrix}
k_2 \\
m_2 \\
n_2 \\
\end{bmatrix} \\
\begin{bmatrix}
k_3 \\
m_3 \\
n_3 \\
\end{bmatrix} \\
\end{array}
\]

\[
\begin{array}{c}
\begin{bmatrix}
k_1 & k_2 & k_3 \\
m_1 & m_2 & m_3 \\
n_1 & n_2 & n_3 \\
\end{bmatrix} \\
\end{array}
\]

\[
\begin{array}{ccc}
c_1 & c_{12} & c_{13} \\
c_{12} & 1 & c_{23} \\
1/c_{13} & 1/c_{23} & 1 \\
\end{array}
\]

Normalization calculation of eigen vector & eigen value:

\[
\begin{array}{c}
\begin{bmatrix}
d \\
f \\
g \\
\end{bmatrix} \\
\end{array}
\]

\[
\sum = 1
\]
Analytical hierarchy process (final step)

- Calculate final priority vector on which decision will be made:

\[
\begin{array}{ccc}
3 \times 3 & 3 \times 1 & 3 \times 1 \\
\begin{array}{ccc}
k_1 & k_2 & k_3 \\
m_1 & m_2 & m_3 \\
n_1 & n_2 & n_3 \\
\end{array} & \begin{array}{c}
d \\
f \\
g \\
\end{array} & \begin{array}{c}
q \\
r \\
t \\
\end{array}
\end{array}
\]

\[=\]

\[
\max \{q, r, t\} \rightarrow \text{choice}
\]
IPRE MODEL

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Model segmentation

- Four sub models:
  1) Constructed real estate for sale
  2) Real estate under construction for sale
  3) Constructed real estate for lease
  4) Real estate under construction for lease

- Five top level criteria common to all four sub models:
  1) Financial strength
  2) Political and legal environment
  3) Project and/or asset characteristics
  4) Strength of the sponsor and developer
  5) Security package

- Subcriteria and their weights differ across sub models
Example of rating criteria for one of the submodels:

- Financial strength
  - Break even
  - Price sensitivity
  - LTV
  - Pre-sales
  - ...

- Political & legal environment
  - Political environment
  - Legal framework

- Asset characteristics
  - Competition
  - Location
  - Design
  - Asset condition

- Strength of sponsor and developer
  - Share in equity
  - Cost overrun
  - Reputation
  - Network strength

- Security package
  - Collateral
  - Income control
  - SPV
IPRE model vs standard AHP methodology

- Usually in AHP options are cardinal variables, while in IPRE they are on ordinal scale (4 slotting grades: from 1 – strong to 4 – weak)

- This has impact on the first step:
IPRE model vs standard AHP methodology

- Pairwise matrix for each subcriterion is not constructed (it is meaningless to compare one slot to the other)
- Alternative approach: for every subcriterion we construct vector that classifies that subcriterion to a certain slot, e.g. one subcriterion classified to 3rd slot:

```
<table>
<thead>
<tr>
<th>Slot</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slot 1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slot 2</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slot 3</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Slot 4</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>
```

- After that process is the same as in standard AHP methodology
Validation

- Low default portfolio – usual backtesting not feasible

- Entire IPRE portfolio of the bank was rated by 3 departments:
  - Sales
  - Underwriting
  - Monitoring

- Consistency of rating distribution among these departments indicate model quality
- Consistency was calculated using ratings matrix and slots drift, e.g:

<table>
<thead>
<tr>
<th>Sales</th>
<th>Credit Risk Underwriting</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
<td>36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slots drift</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>69%</td>
</tr>
<tr>
<td>+1 or -1</td>
<td>29%</td>
</tr>
<tr>
<td>+2 or -2</td>
<td>2%</td>
</tr>
<tr>
<td>+3 or -3</td>
<td>0%</td>
</tr>
</tbody>
</table>
Conclusion

- **Strengths:**
  - Simple
  - Interpretable
  - Usage of domain experts’ knowledge

- **Weaknesses:**
  - Time consuming
  - Decision fatigue (drop of concentration)
  - Correlation between subcriteria (threshold for correlation?)
References


## Contacts

<table>
<thead>
<tr>
<th>Name</th>
<th>Email</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>