INCORPORATING LIFECYCLE AND ENVIRONMENT IN LOAN-LEVEL FORECASTS AND STRESS TESTS

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What are Scores for?

- Improving profitability
  - by
  - Determining the risk of incomplete repayment or reduced revenue
    - by
    - Estimating the risk of default or prepayment
      - by
      - Ranking risk
        - by
        - Computing a score

- This is the assumed hierarchy under which scores are created.
The connection between cut-off score and PD is tenuous.

We can be much more accurate in pricing if we start with a real PD forecast.
Using Scores: Setting Pricing

Score Distribution

Rejects

Cut-off Score

Accepts

Score
The score distribution shifts with the economy.

- The score-odds relationship shifts with the economic cycle and credit cycle.
- Cut-off score is usually adjusted intuitively.
Credit Risk has a group that builds scores that rank-order.
Model Audit validates the scores rank-ordering ability.
Credit Policy sets the cut-off score to control risk (odds of default).
Finance creates a profit model that assumes a certain odds of default.

This structure fails by design.
What we needed from the start was a probability model.
Any score provides that in-sample, but how do we produce it out of sample?
MODELING PROBABILITIES
Drivers of Performance

- **Origination**: Consumer & loan information.
- **Vintages**: Biases for specific cohorts.
- **Lifecycle**: Changes with age of the loan.
- **Behavior**: As the loan matures, observed loan performance can refine the predictions.
- **Account Management**: Changes made to individual loans.
- **Environment**: Changes in the economy or portfolio management.
- **Loan Idiosyncratic Effects**: Every loan is unique.
Age-Vintage-Time (A-V-T) Dependency

<table>
<thead>
<tr>
<th>Information Type</th>
<th>...is a function of</th>
</tr>
</thead>
<tbody>
<tr>
<td>Originations</td>
<td>Recent history or loan (i) up to origination date (v)</td>
</tr>
<tr>
<td>Vintage</td>
<td>Vintage origination date (v)</td>
</tr>
<tr>
<td>Lifecycle</td>
<td>Age of the loan (a)</td>
</tr>
<tr>
<td>Behavior</td>
<td>All time since origination (v, v+1, ..., t=v+a)</td>
</tr>
<tr>
<td>Account Management</td>
<td>Time since event (t_e)</td>
</tr>
<tr>
<td>Environment</td>
<td>Recent time (t-n, ... t-1, t)</td>
</tr>
<tr>
<td>Loan Idiosyntricities</td>
<td>Loan (i)</td>
</tr>
</tbody>
</table>

- Because \( a = t - v \), measurements from one information source will be contaminated by other sources:
  - Behavior is dependent on Lifecycle and Environment.
  - The linear components of any vintage, age, or time dependencies cannot be uniquely measure.
Modeling Implications

- From the Age-Period-Cohort literature, we know that *linear* effects in age, vintage, and time cannot be uniquely determined regardless of the model used.

- For short data sets (such as a typical 2-year scoring dataset), the macroeconomic environment by time and credit risk by vintage are mostly linear trends.

- Coefficients estimated against the linear components of macroeconomic variables and scoring factors will be unstable, i.e. *adding macroeconomic factors to scoring models rarely works.*
1. Use the Age-Vintage-Time structure found in APC, but in a loan-level model.

\[
\log \left( \frac{p_i(a,v,t)}{1 - p_i(a,v,t)} \right) = f(a) + g(v) + h(t), \quad a = t - v
\]

Make an explicit choice about trend allocation between \(f(a), g(v),\) and \(h(t)\) to obtain a unique solution.

1. (Optional) Fit \(h(t)\) to macroeconomic data. Retrend lifecycle and credit risk trends to compensate.

2. Introduce either origination or behavioral scoring factors to estimate score.
A standard approach to defining Good and Bad accounts for logistic-style regression without inclusion of temporal information.
Data for V-A-T Decomposition Methods

- All in-sample loans are included in analysis with emphasis on time of Event occurrence.
- Goods are simply accounts that were not observed to go bad. They are included as censored accounts in the estimation process.
1. Create Loan-level Age-Vintage-Time Model

Put an APC structure into a loan-level GLMM framework.

\[
\log\frac{p_i(a, v, t)}{1 - p_i(a, v, t)} = \alpha_0 + \alpha_1 a + \sum_{j=2}^{N_a-1} \alpha_j O_j(a) + \beta_1 v + \sum_{j=2}^{N_v-1} \beta_j O_j(v) + \sum_{j=2}^{N_t-1} \gamma_j O_j(t)
\]

- 1 constant term, 2 linear terms, and nonlinear terms in \(a\), \(v\), and \(t\). No explicit cross-terms.
- \(O_i\) are orthogonal basis functions.
- \(N_a, N_v, \text{ and } N_t\) are the number of observations in \(a, v, \text{ and } t\).
- Assumes environmental trend = 0.
Recast as functions $F(a), G(v), H(t)$ as

\begin{align*}
F(a) &= \alpha_0 + \alpha_1 a + \sum_{j=2}^{N_a-1}\alpha_j O_j(a) \\
G(v) &= \beta_1 v + \sum_{j=2}^{N_v-1}\beta_j O_j(v) \\
H(t) &= \sum_{j=2}^{N_t-1}\gamma_j O_j(t)
\end{align*}
- Regional US auto loan portfolio, ~$400 mln receivables.
- We modeled PD, EAD, LGD, and attrition to predict losses.
Credit Risk Function Example

Small Auto Loan Portfolio

PD

Vintage Function

Vintage

Small Auto Loan Portfolio

Environmental Function

PD

Time

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2. Fit to Macroeconomic Data

Create OLS fit of $H(t)$ to macroeconomic factors $E_i(t)$ with an explicit trend term. Possibly use a constrained fit so the long-run trend = 0.

$$H(t) = c_0 t + \sum_{i=1}^{N} c_i E_i(t) + \epsilon$$

- We have $T_0$ observations of function $H(t)$.
- We have $T_h$ measurements of $n$ economic factors $E_i(t)$ prior to the start of the performance data.
- Total length of economic data is $T = T_0 + T_h$. 

$-(T_h - 1)$
Create a macroeconomic model

- Model just the environmental function with macroeconomic factors. (Macroeconomic adverse selection impacts on vintage quality is a separate effect.)
- Use the correct transformations: log-ratio, not % change; log interest rates; etc.
- Allow for a linear component.

|                           | Estimate | Std. Error | t value | Pr(>|t|) |
|---------------------------|----------|------------|---------|----------|
| (Intercept)               | 5.908513 | 1.250051   | 4.727   | 3.31E-06 |
| t                         | -0.009547| 0.002535   | -3.765  | 0.000195 |
| Unemp.lwMovingAvg.Lm6.W3  | 2.050558 | 0.535645   | 3.828   | 0.000153 |
| HPI.lwLogRatio.L5.W24     | -2.262367| 1.039257   | -2.177  | 0.030153 |
| Housing.Starts.lwLogRatio.L8.W15 | -0.306789 | 0.047639 | -6.44   | 3.94E-10 |

Residual standard error: 0.434 on 351 degrees of freedom
(12 observations deleted due to missingness)

Multiple R-squared: 0.3918, Adjusted R-squared: 0.3848
F-statistic: 56.52 on 4 and 351 DF, p-value: < 2.2e-16
Retrend the environmental function

Multivariate Forecast of Environmental Function for All Products

- Backward extrapolation
- In-sample

Predicted Environmental Function

Date

Compensate for the trend in \( H(t) \).

\[
F'(a) = F(a) + c_0 t \\
G'(v) = G(v) + c_0 t \\
H'(t) = H(t) - c_0 t = \sum_{i=1}^{N} c_i E_i(t) + \varepsilon
\]

The linear specification problem is NOT unique to APC models. In fact, it appears to be the cause of scores constantly being recalibrated.
Counter-Adjustments to Lifecycle and Credit Risk

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3. Fit the Score

- Using GLM and monthly performance data, use \( F(a) + H(t) \) as a fixed offset.
- The score is estimated as

\[
\text{glm}(\text{Bad} \sim 1 + \text{offset}(F+H) + \text{factor1} + \text{factor2} + \ldots)
\]

- At this point, the factors can be either origination or behavior variables without destabilizing the probability model.
“Offset” captures the population odds. The rest of the model is the idiosyncratic variation – a score.

\[
\text{glm(formula = Bad ~ 1 + offset(offset) + Bureau.Score + log(LTV) + Subcategory + Log.Dep.Bal + Term, family = binomial, ...)}
\]

Deviance Residuals:

-0.2726 -0.0276 -0.0133 -0.0042  4.7835

Coefficients:

| Estimate  | Std. Error  | z value | Pr(>|z|) |
|-----------|-------------|---------|----------|
| (Intercept) | 10.306961   | 1.367256 | 7.538    | 4.76e-14 *** |
| Bureau.Score | -0.012886   | 0.001830 | -7.043   | 1.88e-12 *** |
| log(LTV)   | 1.203598    | 0.546832 | 2.201    | 0.0277 * |
| SubcategoryNew | 0           |          |          |       |
| SubcategoryUsed | 0.133842   | 0.312062 | 0.429    | 0.6680 |
| Log.Dep.Bal | -0.918796   | 0.187975 | -4.888   | 1.02e-06 *** |
| Term       | -0.017266   | 0.008915 | -1.937   | 0.0528 . |

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The glm-avt result is still a “score” if you exclude the offset term.

- It can be scaled to match scores currently in use.
- It will be more robust to changes in the economy – recalibrate much less often.
- It can incorporate measured adverse selection for existing loans.
- It can be built across both older and more recent data history to capture more consumer dynamics.
$p_i$ is the complete probability including the offset term to provide population odds over time.

- It is conditional on age of the loan, "score", and scenario for the environment.
- It is available for each month, arbitrarily far into the future.
- It can be aggregated over any desired time window to predict total probability.
Scoring with and without Offset

The scoring offset = F(a) + H(t)

ROC curve for Bad ~ 1 + Open.Score + Subcategory + Source + Log.Deposit.Bal + log(LTV) + offset(offset)

Auto Model with offset. KS = 0.49. Gini = 0.62
Auto Model without offset. KS = 0.42. Gini = 0.56

False positive rate or (1 − Specificity)
True positive rate or Sensitivity

The scoring offset = F(a) + H(t)
Out-of-Sample Validation

ROC curve for Bad \sim 1 + Open.Score + Subcategory + Source + Log.Deposit.Bal + \log(LTV) + \text{offset(offset)}

Auto

Out-of-Sample. KS = 0.49. Gini = 0.62
In-Sample. KS = 0.5. Gini = 0.63
Out-of-Sample Validation, with and without Offset

ROC curve for Bad ~ 1 + Open.Score + Subcategory + Source + Log.Deposit.Bal + log(LTV) + offset(offset)

Auto
- Out-of-Sample with offset. KS = 0.49. Gini = 0.62
- Out-of-Sample without offset. KS = 0.4. Gini = 0.53

False positive rate or (1 − Specificity)
True positive rate or Sensitivity

Auto
Out-of-Time Model Validation

ROC curve for Bad ~ 1 + Open.Score + Subcategory + Source + Log.Deposit.Bal + log(LTV) + offset(offset)

Auto

Out-of-Time. KS = 0.47. Gini = 0.6

In-Time. KS = 0.48. Gini = 0.61

False positive rate or (1 − Specificity)

Auto

True positive rate or Sensitivity
Out-of-Time Validation, with and without Offset

ROC curve for Bad ~ 1 + Open.Score + Subcategory + Source + Log.Deposit.Bal + log(LTV) + offset(offset)

Out-of-Time with offset. KS = 0.47. Gini = 0.6
Out-of-Time without offset. KS = 0.37. Gini = 0.47
Forecasting

- Use lifecycle and an environmental scenario to create the offset.
- The model predicts loan-level, monthly probabilities.
- Integrate over the product lifetime for pricing or a fixed window for account management.
- Also works for behavior scoring.
Stress Testing & Capital

- No additional modeling is required for stress testing.
- Insert an extreme scenario for the future $H(t)$ and aggregate the probabilities over the desired forecast horizon.
- Through the cycle PDs can be obtained the same way, by setting $H(t) = H_{TTC}$.
- No difference between stress testing and capital.
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