Banking System in Crisis: An Economic Capital Viewpoint

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Introduction

how do advanced economic capital models perform in times before and during a financial crisis?
Expected and Unexpected Loss

- Total losses experienced on a loan portfolio by a financial institution depend on the number of defaults from time to time and their severity.
- The losses that a bank expects to suffer in a given year are known as Expected Losses (EL).
- Losses that exceed the level of expected losses are known as Unexpected Losses (UL).

Figure: Decomposition of Loss into Expected and Unexpected[14]
Expected and Unexpected Loss

- how much capital should a bank hold for being protected from such peaks in losses in an economically efficient way?
- Estimate the probability loss distribution for the given portfolio
- Estimate the loss for a given confidence interval

Figure: Value-at-Risk Approach[14]
Economic Capital

the capital used to provide a cushion against unexpected losses at a specified level over a time horizon is known as economic capital

○ The economic capital viewpoint reflects the aim to measure potential changes in the economic value of assets and liabilities
○ It internally assesses capital position in relation to total risk (market risk, credit risk, concentration risk etc)
○ Is often calculated as Value at Risk
○ Is the generally accepted measurement to assess capital during the ICAAP process
○ Is used for calculation of risk adjusted profitability estimates such as RAROC, RARORAC, EVA
We examine a Corporate Loan portfolio and focus on quantitative approaches that are available for measuring credit and concentration risk.

These approaches try to capture the entire probability distribution of potential losses.

The most associated risk metrics are value-at-risk (\(\text{VaR}\)) and expected shortfall (\(\text{ES}\)).

**VaR**

a measure of potential losses at a chosen confidence level over a defined time horizon (lacks subadditivity).

**ES**

the conditional expectation of loss given that the loss is beyond the VaR level (subadditive measure).
Existing Methodologies II

- The credit migration approach to Value-at-Risk
  - J.P. Morgan’s CreditMetrics[6, 13] is based on the probability of moving from one credit quality to another, including default, which is used to valuate a firm’s assets
  - McKinsey’s CreditPortfolioView[16] where the probabilities of default are a function of macro-variables

- The option pricing approach to VaR as in CreditPortfolioManager by KMV, where the actual probability of default for each obligor is derived based on a Merton type models[12]

- We focus on the actuarial approach as proposed by Credit Suisse’s CreditRisk+[1, 10] model
CreditRisk+

- Default risk is modeled as opposed to credit migration
- There is no dependence between the default risk and the capital structure of the firm
- It enables us to analytically compute the portfolio loss distribution without resorting to Monte Carlo simulations
- PD is modeled as a linear combination of sector default random variables that are gamma distributed:

\[ p_i^s = p_i(\omega_0 + \sum_{k=1}^{K} w_{ki} S_k) \]

where \( w_{ki} \) are the weights of each sector that affects the probability of default, and \( S_k \) are the gamma distributed sector random variables with a mean of 1 and variance \( V[S_k] = \sigma_k^2 \).
CreditRisk+ Extensions I

- The original method used Panjer recursions[15] which are numerically unstable for large loan portfolios - Giese[5] improved upon that by using a recursive computation of exponential and logarithmic polynomials (see also [7])

- The original model assumes that the sector default rates are independent, but in reality these are affected by macroeconomic indicators and are in some part correlated. To address that:
  - In Bürgisser et al[2] a single sector model is calculated, with an adjusted portfolio default rate standard deviation according to sector correlations: if we denote by $EL_i$ the expected loss for sector $i \in \{1, \ldots, K\}$, then the relative default frequency $\sigma^2$ is computed by:

  $$\sigma^2 = \frac{\sum_{k=1}^{K} \sigma_{S_k}^2 EL_k^2 + \sum_{k \neq l} \text{corr}(S_k, S_l) \sigma_{S_k} \sigma_{S_l} EL_k EL_l}{EL^2}$$

  (1)

  - In Giese[5] the sector default rates are conditioned on a single gamma distributed random variable, which induces a uniform level covariance between sectors, and have the form $S_k = \sigma_k^2 (Y_k + \eta_k^2)$
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\sigma^2 = \sum_{k=1}^{K} \frac{\sigma^2_{S_k} EL^2_k + \sum_{k \neq l} \text{corr}(S_k, S_l) \sigma_{S_k,S_l} EL_k EL_l}{EL^2} \tag{1}
$$

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$$

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    \sigma^2 = \frac{\sum_{k=1}^{K} \sigma^2_{S_k} EL_k^2 + \sum_{k \neq l} \text{corr}(S_k, S_l)\sigma_{S_k,S_l} EL_k EL_l}{EL^2}
    \]  

  - In Giese\cite{5} the sector default rates are conditioned on a single gamma distributed random variable, which induces a uniform level covariance between sectors, and have the form $S_k = \sigma^2_k (Y_k + \hat{Y})$
The hidden gamma model of Giese[5] is consistent with the common sense narrative that a macroeconomic factor induces correlation to sector default rates.

…but at the same time, the covariance structure the model can describe is restricted: the common risk factor is \( \hat{Y} \sim Gamma(\theta, 1) \), and in [5] it is shown that for sector variances \( \sigma_k^2 \), bounds on \( \theta \) are induced:

\[
0 \leq \theta \leq \min_k \left\{ \frac{1}{\sigma_k^2} \right\}
\]

which lead to considerable restrictions to the covariance structure the model can describe[8].
In [8] the model was generalized by giving up the condition that the coefficient of $Y_k$ must be the same as that of $\hat{Y}$, ie

$$S_k = \delta_k Y_k + \gamma_k \hat{Y}$$

which allows a wider range of sector covariance structures

Fischer & Dietz in [4] extended the model further by considering several additive $T_L$ background factors
The Common Background Vector Model I

Following [4], the default indicator of counterparty A is approximated by the following Poisson variable:

\[ \lambda_A^S = p_A(w_{A0} + w_{A1} \hat{S}_1 + \cdots + w_{AK} \hat{S}_K + w_{A,K+1} T_1 + \cdots + w_{A,K+L} T_L) \]

where the sector impact is determined by weights \( w_{Aj} \) and \( w_{A0} = 1 - w_{Ak} - \cdots - w_{A1} \), and where we have \( K \) dependent sector variables \( \hat{S}_K \) and \( L \) \( T_L \) common factors that connect the underlying independent \( K \) sector variables \( S_1 \ldots S_K \):

\[ \hat{S}_K = \delta_k S_k + \sum_{l=1}^{L} \gamma_{lk} T_l \]

with \( S_k \sim \Gamma(\theta_k, 1) \) and \( T_l \sim \Gamma(\hat{\theta}_l, 1) \) for \( k = 1, \ldots, K \) and \( L \) weights \( w_{A,K+I} = \sum_{k=1}^{K} w_{Ak} \gamma_{lk} \) for \( I = 1, \ldots, L \).
The theoretical sector VCV derives as:

\[ E[\hat{S}_k] = \delta_k \theta_k + \sum_{l=1}^{L} \gamma_{lk} \hat{\theta}_l \]

\[ Var[\hat{S}_k] = \delta_k^2 \theta_k + \sum_{l=1}^{L} \gamma_{lk}^2 \hat{\theta}_l \]

\[ \text{cov}(\hat{S}_i, \hat{S}_j) = \sum_{l=1}^{L} \gamma_{li} \gamma_{lj} \hat{\theta}_l \]

The original CreditRisk+ model then can be applied where the parameters of the model have been appropriately substituted by \( \theta_j, \theta_l \) and \( \delta_j \), where \( j = 1, \ldots, K + L \) and \( l = 1, \ldots, L \) as described in [4].
The unknown parameters are chosen such that we minimize the distance between the observed and modeled VCV matrix, under some constraints:

$$\min_{\hat{\theta}_k, \theta_l, \delta_k, \gamma_{lk}} \sum_{k=1}^{K} (\sigma^2_k - \delta_k \hat{\theta}_k - \sum_{l=1}^{L} \gamma^2_{lk} \theta_l)^2 + \sum_{k=1}^{K} \sum_{j=1}^{k-1} (\sigma_{kj} - \sum_{l=1}^{L} \gamma_{lk} \gamma_{lj} \theta_l)^2$$

s.t. (2)

$$\delta_k \theta_k + \sum_{l=1}^{L} \gamma_{lk} \hat{\theta}_l = 1 \quad \forall k \in \{1 \ldots K\}$$

(3)

$$\omega_{A,K+l} \geq 0 \quad \forall A \in A \text{ and } \forall l \in \{1 \ldots L\}$$

(4)

$$\theta_k, \hat{\theta}_l, \delta_k \geq 0 \quad \forall k \in \{1 \ldots K\} \text{ and } \forall l \in \{1 \ldots L\}$$

(5)

where by $A$ we denote the set of all counterparties.
Implementation

- Results were derived from two CreditRisk+ implementations (i) the original CreditRisk+ as adjusted in [2], i.e. using the equation (1) to determine sector correlation, (ii) the CBV model as described in the previous section.

- Both models are fit for quickly computing loss distributions via deriving their PGF with minimal pre-computation, using FFT as described in [11] for implementation (i), and the recursion scheme introduced by Giese for implementation (ii) following [4].

- Implementation (i) runs instantly, and a multithreaded Java implementation (parallelizing computations for each sector) for (ii) has runtime of a few seconds to a few minutes, for corporate portfolios ranging from a few thousand to hundreds of thousands of loans (having solved the minimization problem in a pre-computation step).
Data and Pre-Computation

- Segments of corporate portfolios (portfolios resulting from mergers were excluded) for the years before and during the economic crisis in Greece
- Exposures ranged from a few Euros to millions of Euros
- To specify the observed VCV matrix, the general rule was to use time series in corporate defaults $T=18$ months before the year in examination, where available
- We restricted ourselves to 23 industry sectors (or factors, in terms of the model) along with an idiosyncratic factor and 3 common background factors (i.e. $L = 3$)
- The empirical VCV matrix may not be positive semidefinite (PSD) which leads to non-convexity problems during the optimization step described in (2). There exist a number of methodologies for such a transformation to the nearest PSD matrix (see e.g. [9, 3])
Empirical Results I

We provide VaR and ES figures for the above mentioned datasets, using implementation (i) (CreditRisk+ as adjusted in [2]), (ii) (the CBV model) plus results relating the economic capital with the regulatory capital associated with each sample/year.

Figure: VaR at 99.9% with different methodologies.
Empirical Results II

VaR Year 1

VaR Year 3

VaR Year 2

VaR Year 4

- CBV
- Bürgisser et al
Empirical Results III

**ES Year 1**
- 90%
- 95%
- 99%

**ES Year 2**
- 90%
- 95%
- 99%

**ES Year 3**
- 90%
- 95%
- 99%

**ES Year 4**
- 90%
- 95%
- 99%
Conclusions

- CreditRisk+ assumes sectors are independent: we implemented and adjusted two methodologies for estimating VaR where this issue is being addressed.

- The first (Burgisser et.al.)[2] is a well-known methodology that attempts to model sector correlation, while the second one (CBV [4]) is a very recent method which attempts to incorporate the actual correlation between each pair of factors in our model.

- The first model overestimates the risk due to the common correlation factor which is driven by particular sectors which are extremely correlated in downturn conditions.

- Numerically stable recursion schemes as introduced in [5] and well-posed optimisation setups (by calibrating our model using PSD vcv matrices[9, 3]) are critical in successful implementations of the CBV model.
Future Research

- Among the next steps in such an analysis is to omit the gamma-distributed sectors and introduce new appropriate distributions, since there is no empirical evidence for such a selection.
- Extreme Value Theory can be used to estimate tail risks, especially for the Expected Shortfall metric.
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