A MEAN-REVERTING MODEL TO CREATE MACROECONOMIC SCENARIOS FOR CREDIT RISK MODELS

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Goals

- Proposed FASB guidelines on loss reserves require lifetime loss forecasts.
- Loan pricing requires a lifetime loss forecast.
- Lifetime loss forecasts require long-range economic scenarios.
- We want a simple way to generate macroeconomic scenarios that start with current conditions and relax onto long-run averages.
Scenarios for Long-term Forecasting

- Use your best macroeconomic assumptions for the first 12 months.
- Relax to the mean over the next 12 months.
- Use the long-term historic average beyond 24 months.
Options for Creating Scenarios

- **Damped extrapolation**
  - Choose an appealing function

- **Mean-reverting process**
  - Can provide both mean and variance
  - Creates PIT and TTC economic capital estimates
Method of Analysis

1. Create a forecast model that accepts macroeconomic scenarios.
2. Collect the macroeconomic sensitivity of the credit risk model into a single index.
3. Calibrate a mean-reverting model to the macroeconomic sensitivity index.
4. Obtain a macroeconomic scenario from economist(s).
5. Overlay the mean-reverting model onto the macroeconomic scenario.
1. Create a Forecast Model

- For lifetime forecasting, choose a model that captures the lifecycle(a), environment(t), and credit risk(v).
- In this example, we use a GLM-AVT model. A loan-level version of an Age-Period-Cohort model.

\[
\log \frac{p_i(a, v, t)}{1 - p_i(a, v, t)} = f(a) + g(v) + h(v), \quad a = t - v
\]
Lifecycle Function Example

Small Auto Loan Portfolio

Age

Lifecycle Function

0.000 0.001 0.002 0.003 0.004

0 12 24 36 48 60 72 84 96 108

0.000 0.001 0.002 0.003 0.004

0 12 24 36 48 60 72 84 96 108
Credit Risk Function Example

Small Auto Loan Portfolio

Vintage Function

Vintage
Small Auto Loan Portfolio

Environmental Function

Time

-2

-1

0

1

2

2. Create a Macroeconomic Sensitivity Index

- The method applies to any model that accepts macroeconomic inputs such that they can be collected into single function:
  - Logistic regression
  - Cox PH
  - GLM
  - Age-Period-Cohort
  - GLM-AVT
  - LR-AVT

- Define \( H'(t) = \sum_{i=1}^{N} c_i E_i(t) + \epsilon_i \)

as the environmental function – the macroeconomic sensitivity index. Assumes \( H'(t) \) is normally distributed.
Through-the-Cycle Estimates

- Extrapolate the macroeconomic sensitivity backward through previous economic environments.
- Compute the TTC values from the fit to macroeconomic data $H'(t)$, not the short amount of observed history $H(t)$.

\[
H(t) = \sum_{i=1}^{N} c_i E_i(t) + \varepsilon
\]
3. Calibrate a Mean-Reverting Model

- Most common is the Ornstein-Uhlenbeck process:

\[ dx_t = \theta(\mu - x_t)dt + \sigma dW_t \]

- Where \( x_t \) is the time series being simulated, \( \mu \) is the long run mean, \( \sigma \) is the deviation about this mean, \( \theta \) is the relaxation rate to the mean, and \( W_t \) is a Wiener process.

- The Vasicek model is an Ornstein-Uhlenbeck process.

- The Ornstein-Uhlenbeck model is the only stationary, Gaussian, and Markovian model possible.
Mean-Reverting Models in Discrete Time

- Our data is always discrete time, and almost always uniformly spaced in time.

- A discrete-time simplification is

\[
\Delta x_t = \theta(\mu - x_t) \Delta t + \varepsilon_t
\]

\[
\mu = d - \frac{\sigma^2}{2\theta}, \quad \varepsilon_t \approx N(0, \sigma)
\]

- Where \(d\) is the drift term. From this, we get

\[
E(x_t) = (1 - e^{-\theta(t-t_0)}) \mu + e^{-\theta(t-t_0)} x_{t_0} \xrightarrow{t \to \infty} \mu
\]

\[
Var(x_t) = \frac{\sigma^2}{2\theta} (1 - e^{-2\theta(t-t_0)}) \xrightarrow{t \to \infty} \frac{\sigma^2}{2\theta}
\]
Mean-Reverting Models with A-V-T

- Model the environmental impacts

\[ \Delta H(t) = \theta(\mu - H(t))\Delta t + \varepsilon_t, \quad \varepsilon_t \approx N(0, \sigma) \]

\[ E(H(t)) = (1 - e^{-\theta(t-t_0)})\mu + e^{-\theta(t-t_0)}H(t_0) \quad \stackrel{t \to \infty}{\longrightarrow} \quad \mu = H_{TTC} \]

\[ Var(H(t)) = \frac{\sigma^2}{2\theta} (1 - e^{-2\theta(t-t_0)}) \quad \stackrel{t \to \infty}{\longrightarrow} \quad \frac{\sigma^2}{2\theta} = \sigma^2_{TTC} \]

- The previous Through-the-Cycle analysis provides \( H_{TTC} \) and \( \sigma^2_{TTC} \).
Connecting PiT and TTC

- The half-time (time to relax half-way to the mean) is
  \[ t_{1/2} = \frac{\ln(2)}{\theta} \]

- If we assume that \( t_{1/2} \) is approximately 1.5 years (based upon Monte Carlo experiments), then
  \[ \sigma^2 = \frac{4}{3} \ln(2) \sigma_{TTC}^2, \quad \mu = H_{TTC} \]

- And we have a fully specified mean-reverting model.
5. Application of Mean-Reverting Model
5. Application of Mean-Reverting Model

![Graph showing time series with labeled axes and markers for history and scenario.]
Conclusions

- This does a reasonable job at creating a distribution of possible futures.
  - Relaxes only the TTC mean.
  - Converges to the TTC variance.

- Can be used to
  - Generate a single mean scenario for forecasting or pricing.
  - Generate extreme scenarios for PIT and TTC economic capital.
Model Extensions

- Markovian processes depend only on the current state.
- Macroeconomic time series show autocorrelation out to 6 to 12 months.
- A more accurate model might include lagged dependencies.
- Such models are significantly more complex.
Contact Us

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