Understanding Differential Cycle Sensitivity for Loan Portfolios

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Context & Background

- At Westpac we have recently conducted a revision of our **Probability of Default models**, an important consideration was how the economy effects point in time default rates.
- We have also been making a number of changes to our **Economic Capital framework** including a review of **Asset Correlation** estimates.

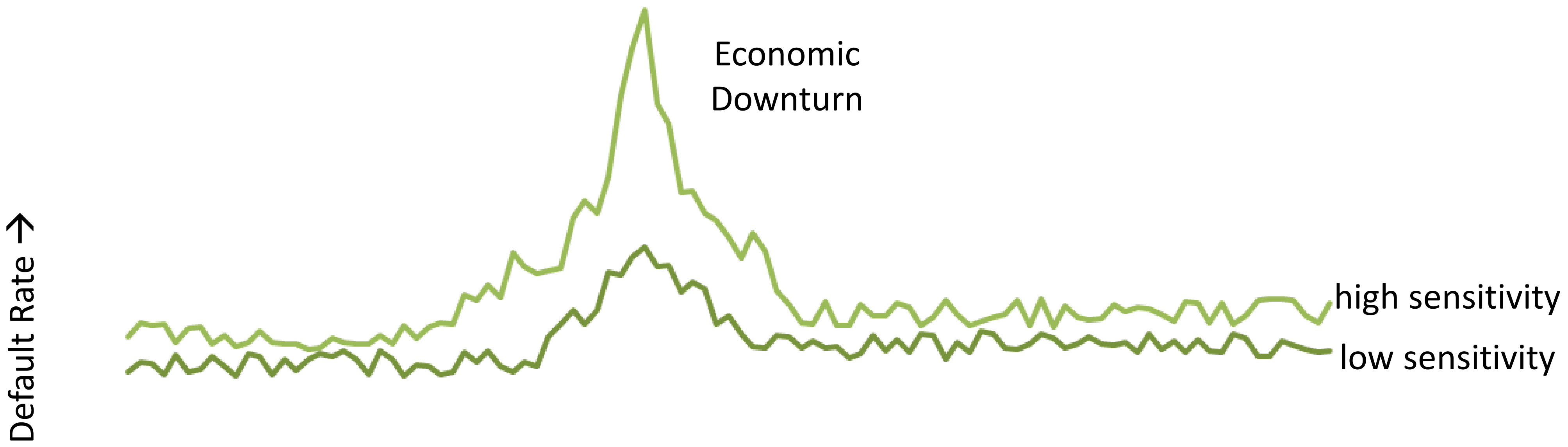
Agenda

- Defining Cycle Sensitivity
- Empirical Analysis
- Interpretation of Results & Conclusions

Defining Cycle Sensitivity

Defining 'Cycle Sensitivity'

Cycle Sensitivity is any measure of the **responsiveness or sensitivity of a credit risk measure** (PD, EAD, LGD) or credit losses to **changes in macro-economic conditions** (aka the economic 'cycle').



Defining 'Cycle Sensitivity'– Drivers of PD

Three drivers of **point in time** default rates are considered in the model…

$$PD = f(S, Z, \varepsilon)$$

S = Credit Quality

(Credit Score)

Z = Economy

(State of the Cycle Indicator)

$$Z > 0 \Longrightarrow good\ economy$$

$$Z < 0 \Longrightarrow bad\ economy$$

ε = Residual Volatility

(Non cycle driven random effects)

Defining ‘Cycle Sensitivity’ - PD Model Structure

The model is parameterized as follows...

$$\mathrm{G}^{-1}(PD) = \alpha + \beta \cdot Z + \varepsilon$$

Defining ‘Cycle Sensitivity’ - PD Model Structure

The model is parameterized as follows...

$$\mathrm{G}^{-1}(PD) = \alpha(S) + \beta(\alpha, \tilde{x}) \cdot Z + \varepsilon \qquad \varepsilon \sim N(0, \sigma(\alpha, \tilde{x})^2)$$

Normally distributed residual

G = Link Function

α = Credit Quality Parameter

Dependent on credit score (S)

β = State of the Cycle co-efficient

Can depend on….

> Credit Quality (α)

> Other Factors ($\tilde{x}$)

Defining ‘Cycle Sensitivity’– Conditional (Point in Time) PD

Conditional (**point in time**) probability of default can be calculated as follows…

$$E(PD|Z) = \int_{-\infty}^{\infty} G(\alpha + \beta \cdot Z + \varepsilon) P_{\varepsilon}(\varepsilon) \partial\varepsilon$$

Conditional PD

probability density ε

$$P_{\varepsilon}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

Defining ‘Cycle Sensitivity’– Basic Idea

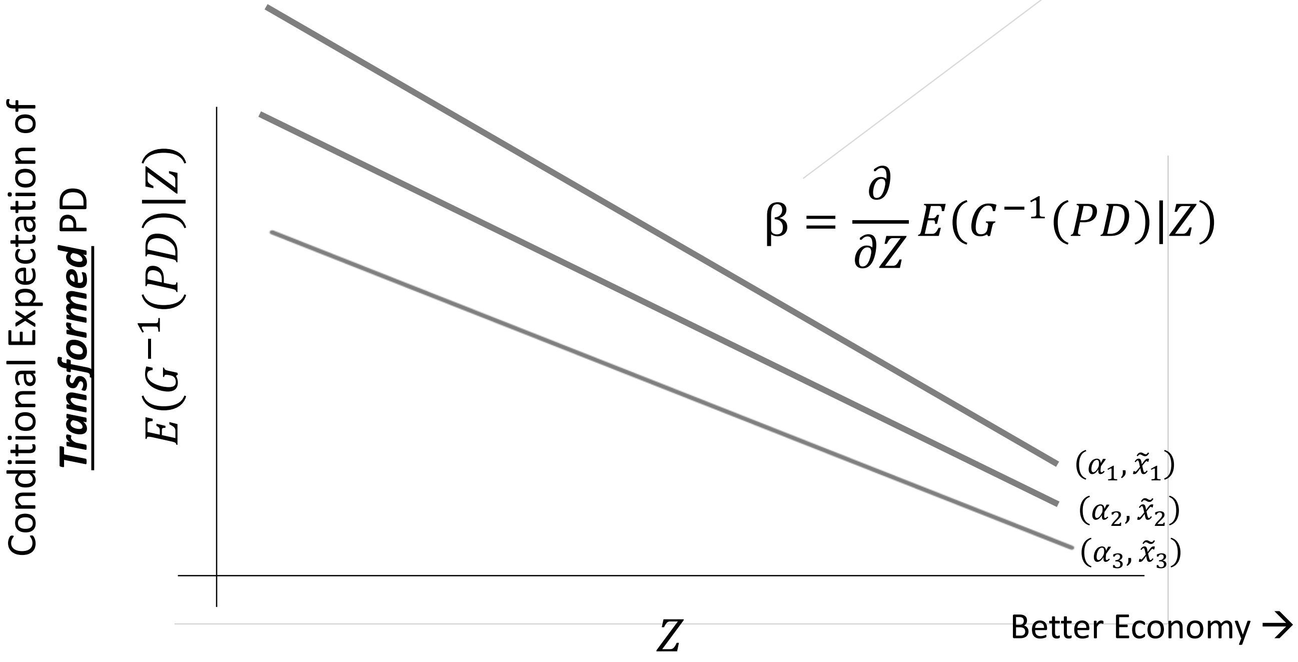
Cycle sensitivity for any credit risk measure can be defined by considering the rate of change (derivative) with respect to Z.

$$Cycle\ Sensitivity\ (f) = \frac{\partial f}{\partial Z}$$

Defining ‘Cycle Sensitivity’- Beta

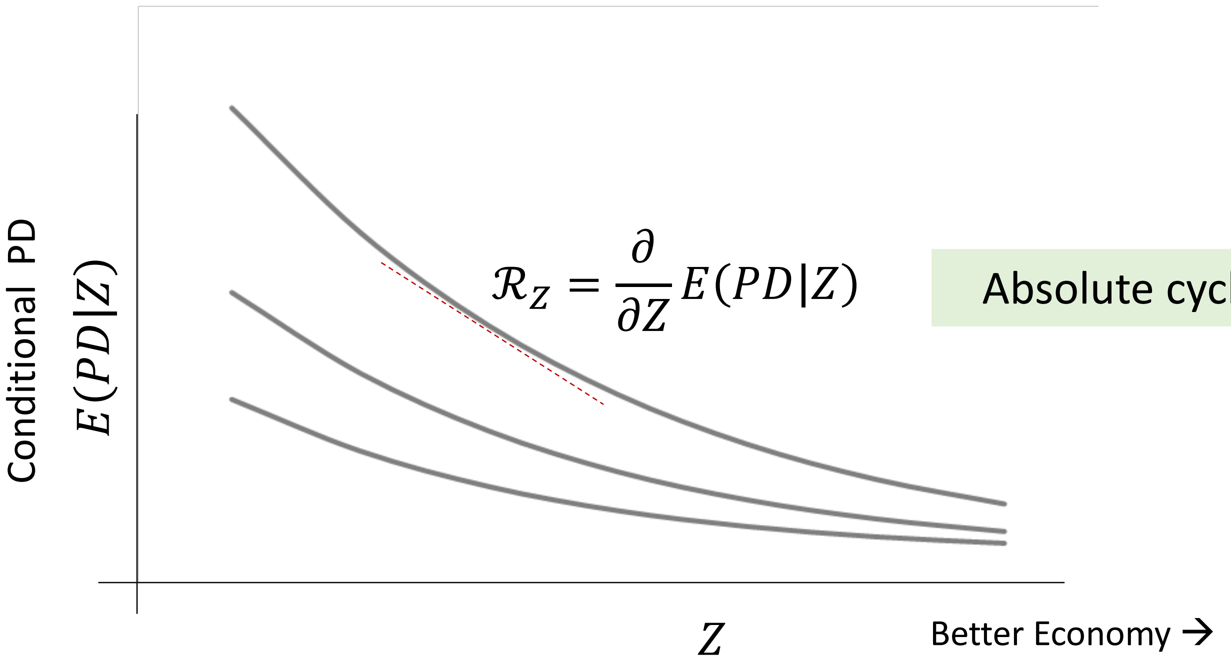
The state of the cycle indicator co-efficient β can be considered a measure of cycle sensitivity measuring the **rate of change of the transformed PD**.

$$E(G^{-1}(PD)|Z) = \alpha + \beta \cdot Z + \overset{=0}{E(\varepsilon)}$$



Defining 'Cycle Sensitivity' - Absolute Cycle Sensitivity

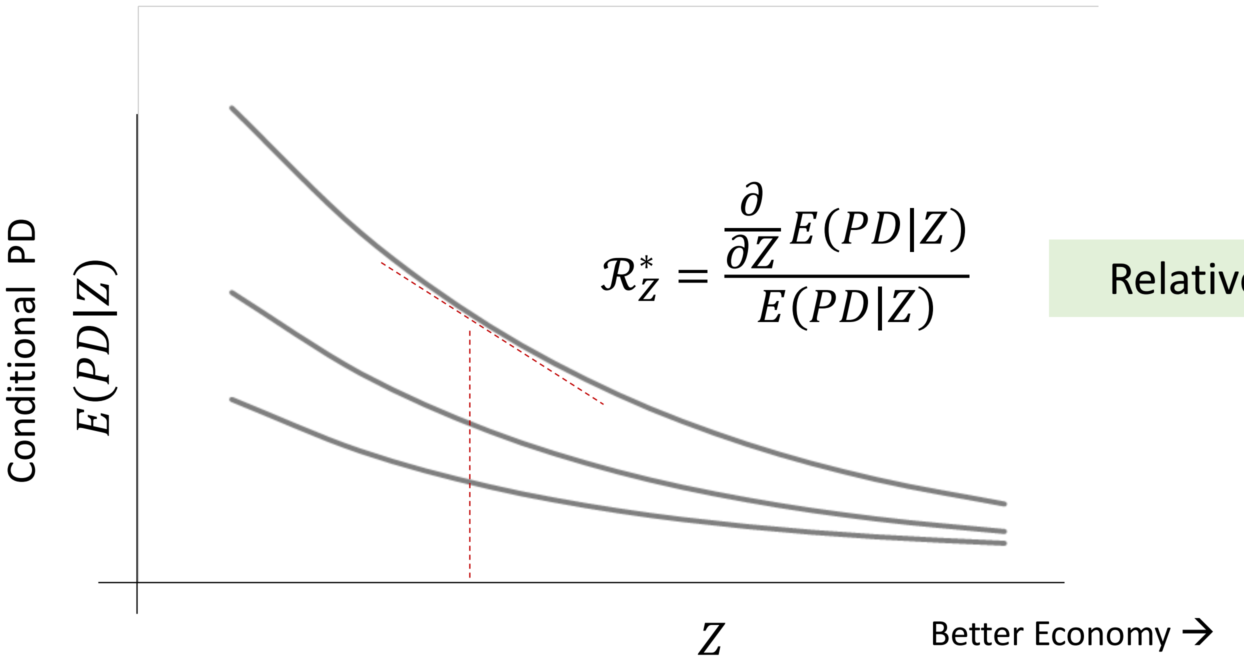
A simple and intuitive definition of cycle sensitivity is the **rate of change of conditional PD with respect to the state of the cycle measure**.



Absolute cycle sensitivity

Defining ‘Cycle Sensitivity’- Relative Cycle Sensitivity

A measure can also be defined by **dividing absolute cycle sensitivity by PD**. This measure provides a measure of **relative change** in PD with respect to the state of the cycle.



$$\mathcal{R}_Z^* = \frac{\frac{\partial}{\partial Z} E(PD|Z)}{E(PD|Z)}$$

Relative cycle sensitivity

Defining 'Cycle Sensitivity' – Example, Log Link Function

Closed form solutions to the cycle sensitivity measures can be defined for some choices of link function.

Natural log link function… $G^{-1}(x) = -log(x)$

$$-log(PD) = \alpha + \beta \cdot Z + N(0, \sigma^2)$$

conditional PD… $E(PD|Z) = exp(-(\alpha + \beta \cdot Z) + \sigma^2/2)$

differentiate… $\mathcal{R}_Z = -\beta \cdot exp(-(\alpha + \beta \cdot Z) + \sigma^2/2)$ Absolute cycle sensitivity

divide by $E(PD|Z)$… $\mathcal{R}_Z^* = -\beta$ Relative cycle sensitivity

Defining ‘Cycle Sensitivity’ – Example, Log Link Function

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differentiate… $\mathcal{R}_Z = -\beta \cdot exp(-(\alpha + \beta \cdot Z) + \sigma^2/2)$

divide by $E(PD|Z)$… $\mathcal{R}_Z^* = -\beta$

* **Warning** – algebraically appealing though PD unbounded!

$$\lim_{Z \to -\infty} E(PD|Z) = +\infty$$

Defining 'Cycle Sensitivity' – Example, Probit Link Function

In general cycle **sensitivity measures are Z-dependent** if the conditional **PD is a bounded function of Z.** Probit or Logit link functions are good options.

Probit link function… $G^{-1}(x) = -\Phi^{-1}(x)$

$$-\Phi^{-1}(PD) = \alpha + \beta \cdot Z + N(0, \sigma^2)$$

$$E(PD|Z) = \Phi\left(-(\alpha + \beta \cdot Z)(1+\sigma)^{-1/2}\right)$$

$$\mathcal{R}_Z = -\beta \cdot (1+\sigma)^{-1/2} \cdot \phi\left(-(\alpha + \beta \cdot Z)(1+\sigma)^{-1/2}\right)$$

$$\mathcal{R}_Z^* = -\beta \cdot (1+\sigma)^{-1/2} \cdot \frac{\phi\left(-(\alpha + \beta \cdot Z)(1+\sigma)^{-1/2}\right)}{\Phi\left(-(\alpha + \beta \cdot Z)(1+\sigma)^{-1/2}\right)}$$

Bounded PD

$$\lim_{Z \to -\infty} E(PD|Z) = 1$$

Asset Correlation

The **Asset Correlation** parameter under a Merton-ASRF framework is one example of a cycle sensitivity measure based on particular model parameterization and underlying model assumptions.

$$\mathrm{PD}(Z) = \Phi\left[\frac{\Phi^{-1}(PD_{TTC}) + \sqrt{\boldsymbol{\rho}} \cdot Z}{\sqrt{1-\boldsymbol{\rho}}}\right]$$

$$Z \sim N(0,1)$$

(Latent Systemic Factor)

The Merton-ASRF model assumes:

- Defaults are triggered by the value of an obligor’s assets falling below the value of their debts.
- All macro-economic or systemic influences on default rates can be described by a normally distributed, un-observable (latent) factor.

Asset Correlation

Our model parameterization reduces to a Merton-ASRF model if a number of simplifying assumptions are introduced (we believe some of these are unrealistic, particularly normality of Z).

Simplifying Assumptions: $Z \sim N(0,1)$ $\quad corr(X,Z) = 0$ $\quad G(x) = -\Phi^{-1}(x)$ $\quad \alpha, \beta, \sigma = constant$

$$\Rightarrow E(PD|Z) = \Phi\left[\frac{const + \sqrt{\boldsymbol{\rho}} \cdot Z}{\sqrt{1-\boldsymbol{\rho}}}\right]$$

$$\boldsymbol{\rho} = \frac{\beta^2 + \sigma^2}{1 + \beta^2 + \sigma^2}$$ **Asset Correlation Estimate**

Advantages of new Measures

We believe the **advantages** of the absolute and relative cycle sensitivity measures over more traditional measures are…

- Can be meaningfully compared across different populations and segments.
- Easily explained and interpreted by non technicians.
- Trends behave more intuitively than other popular measures (e.g. asset correlation)

Empirical Analysis

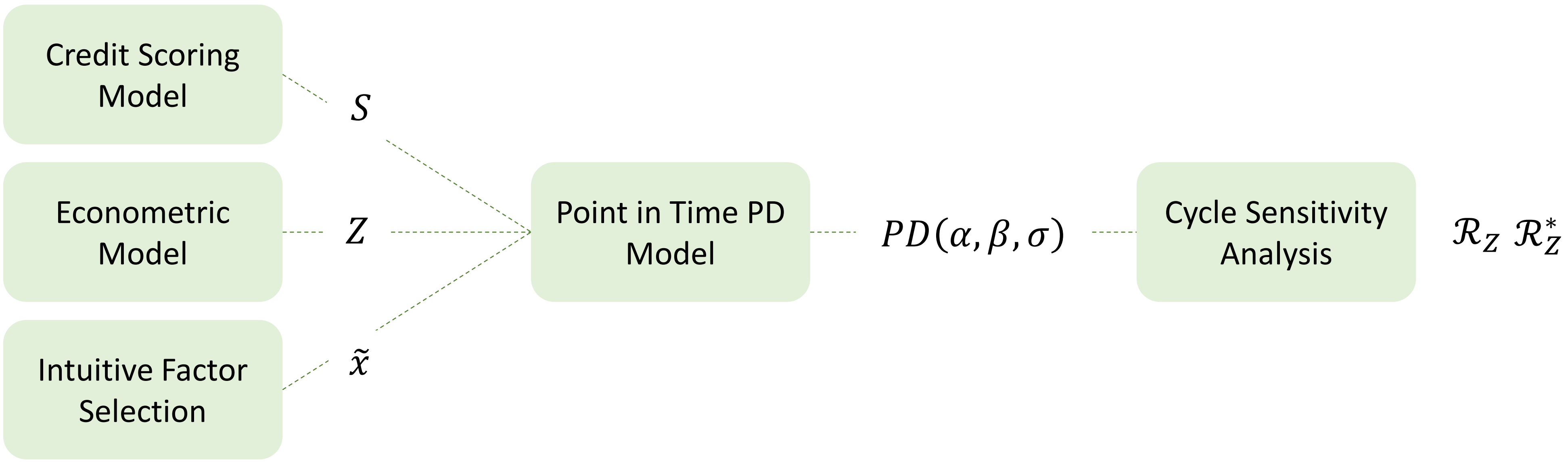
Portfolios

PD model were developed for the following portfolios….

- Mortgages
- Personal Loans
- Credit Cards

PD Model Structure

A basic outline of model development process is shown below...



PD Model Structure – Point in Time PD

The point in time model was developed using a **piecewise linear probit-model** where probit-transformed observed default rates were regressed against the state of the cycle indicator within a set of discrete, mutually exclusive segments.

$$-\Phi^{-1}[PD_i] = \alpha_i + \beta_i \cdot Z + N\left(0, \sigma_i^2\right)$$

i denotes segment

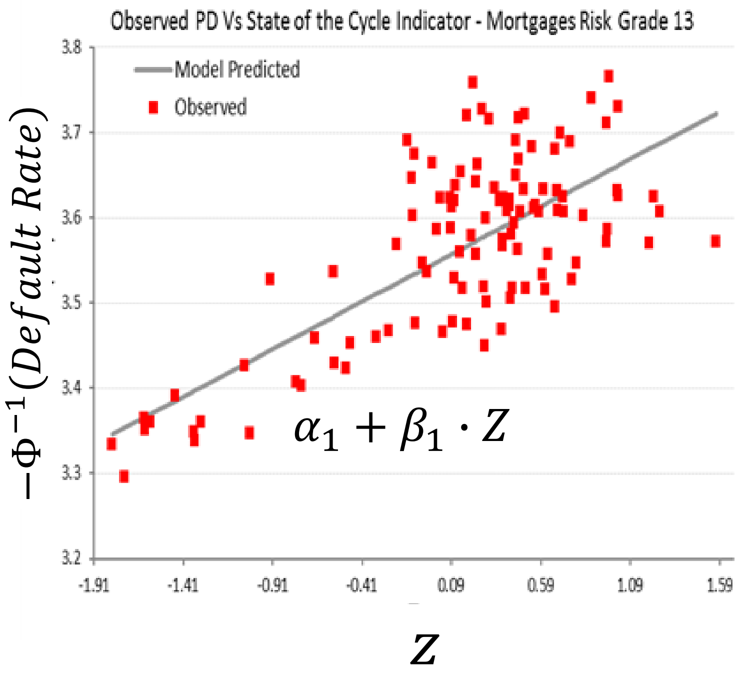
Key segments investigated were…

- Risk Grade (Credit Score Banding)
- LVR
- Product

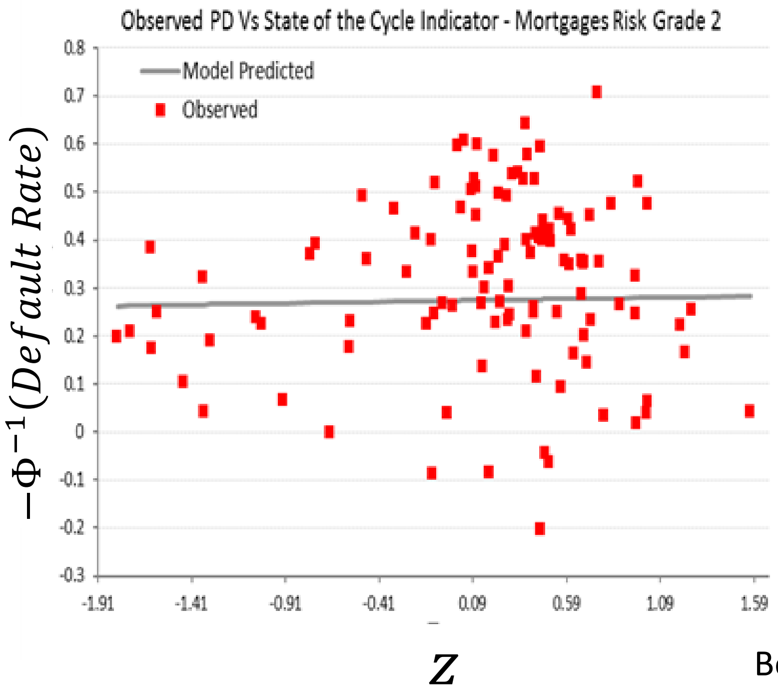
Risk Grade Segmented Model

The cycle sensitivity parameter (beta) was found to have a dependency on credit score for consumer loan products (mortgages and personal loans). Default rates in the highest risk grades (highest average PD) showed no significant dependency on the state of the cycle indicator, whereas the lowest risk grades showed greatest sensitivity.

Low Risk Grade



High Risk Grade



$\alpha_{13} + \beta_{13} \cdot Z$

Better Economy →

Risk Grade Segmented Model Results – Alpha

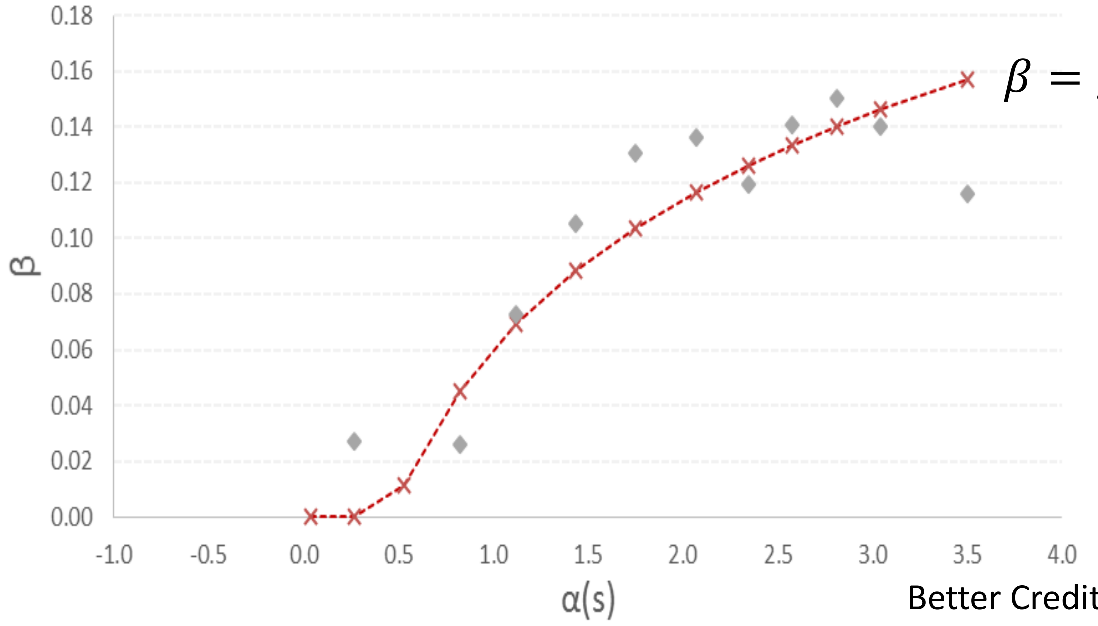
The strong monotonic relationship is not surprising given credit score groupings defining risk grades were chosen to ensure a relatively high degree of separation in performance for adjacent grades.

$$G^{-1}(PD) = \alpha(S) + \beta \cdot Z + N(0, \sigma^2)$$

Risk Grade Segmented Model Results – Beta

A **continuous relationship was fit between $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$** to simplify the model structure and reduce degrees of freedom…

Mortgages β vs α



$$\beta = g(\alpha) = c_1 + c_2 \cdot log(\alpha)$$

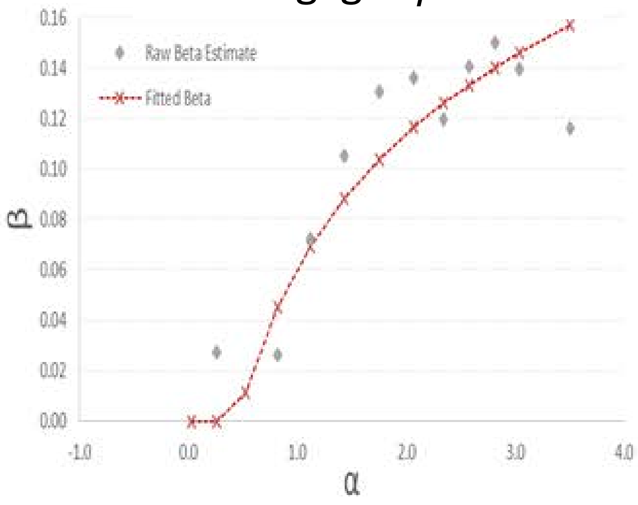
Better Credit Quality →
(lower PD)

Risk Grade Segmented Model Results – Beta

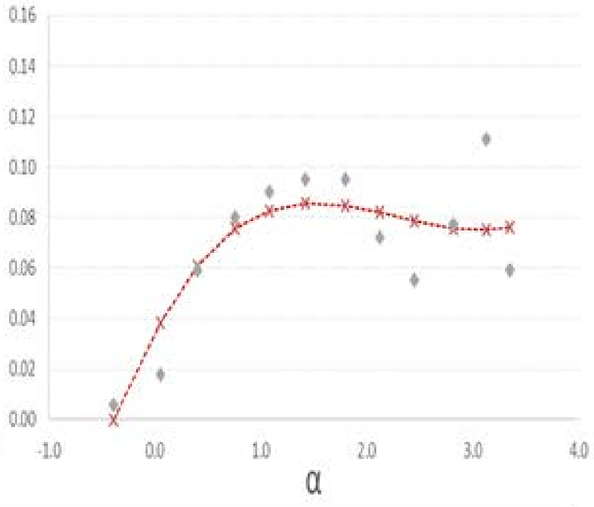
The cycle sensitivity parameter was found to have a dependency on credit quality for all portfolios tested.

$$G^{-1}(PD) = \alpha + \boldsymbol{\beta(\alpha, product)} \cdot Z + N(0, \sigma^2)$$

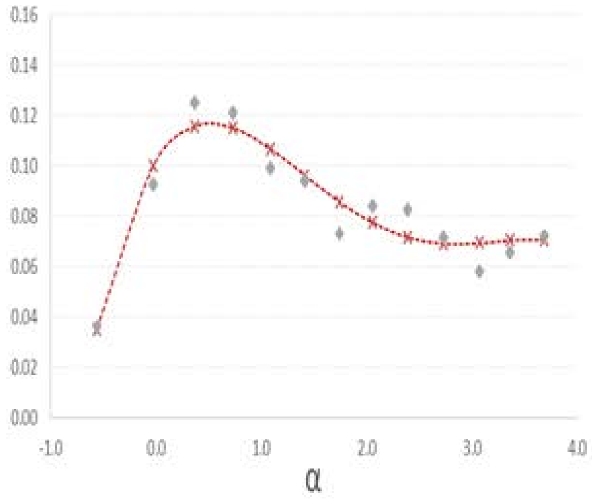
Mortgages β vs α



Personal Loans β vs α

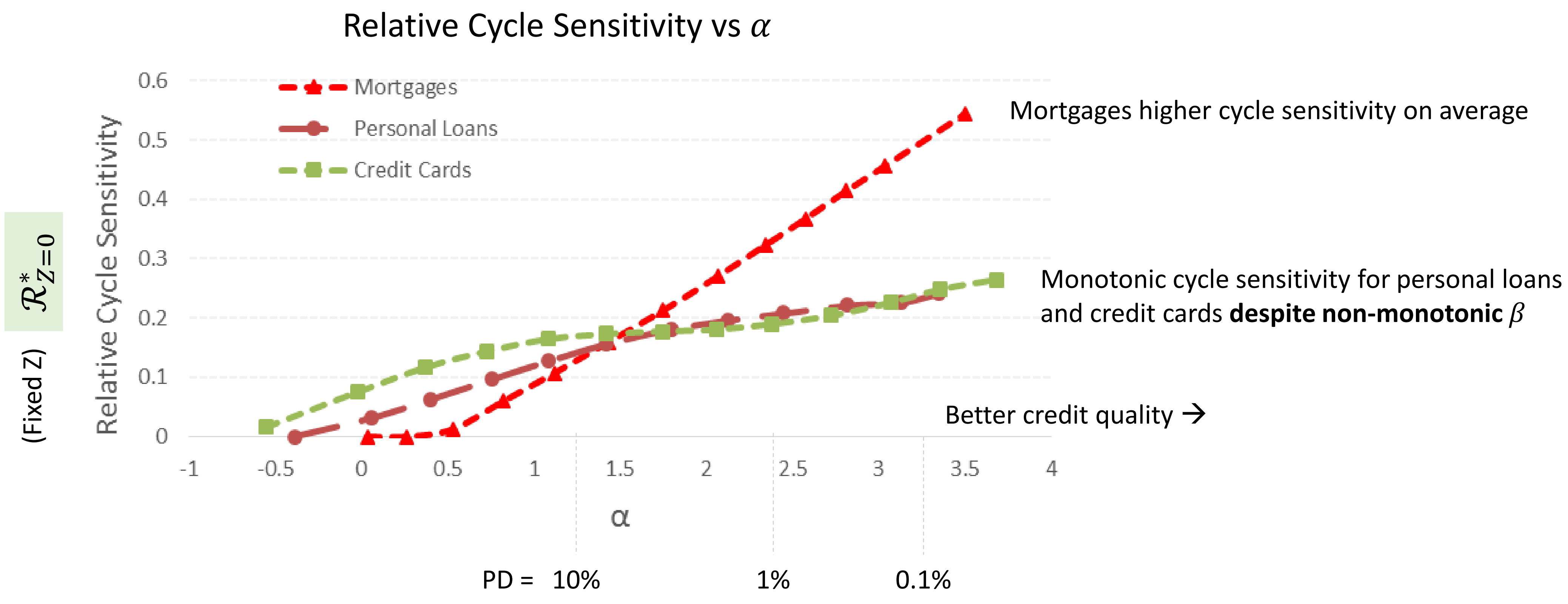


Credit Cards β vs α



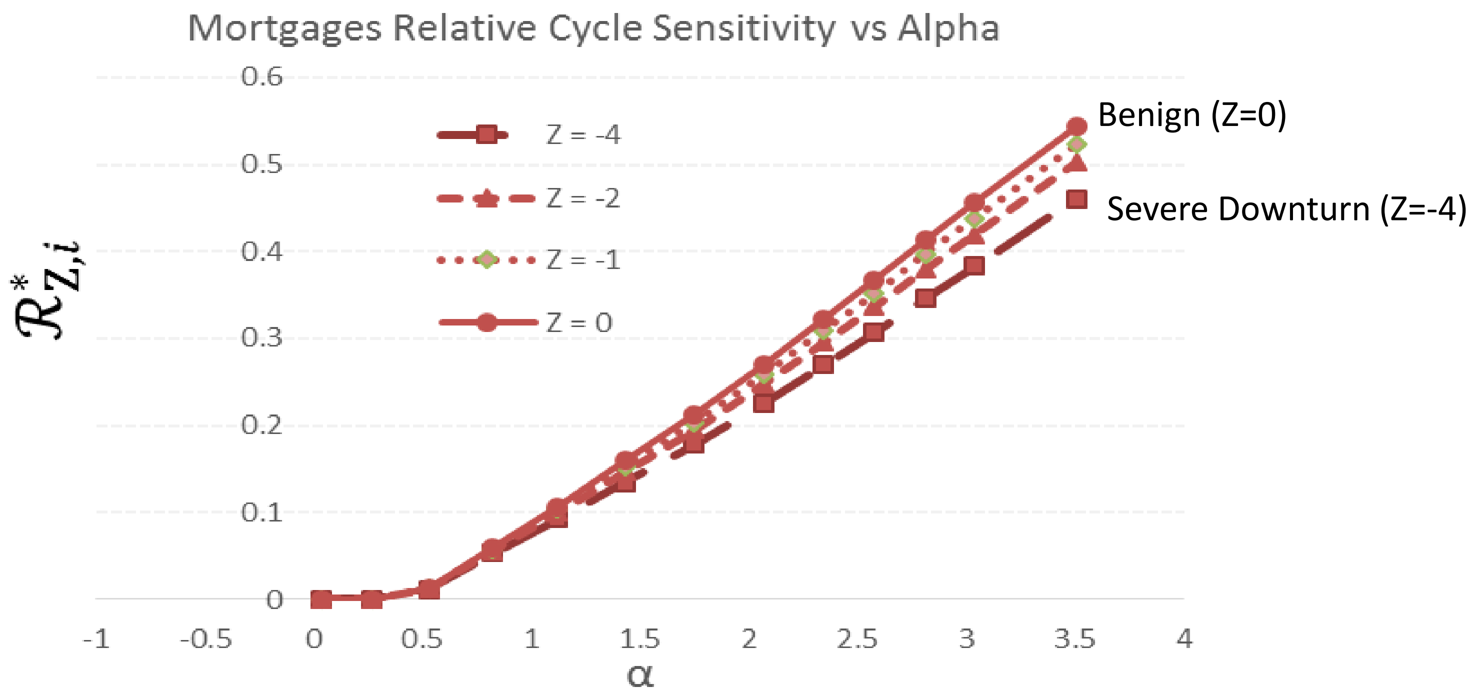
Risk Grade Segmented Model Results – Relative Cycle Sensitivity

Relative cycle sensitivity was found to increase with increasing credit quality (lower risk segments more sensitive). Mortgages were more sensitive than credit cards and personal loans.



Risk Grade Segmented Model Results – Cycle Sensitivity Z-dependence

Relationships between cycle sensitivity measures and credit quality are qualitatively the same regardless of state of the cycle (value of Z). Relative sensitivity decreases in a downturn as the denominator (conditional PD) increases.



LVR Model

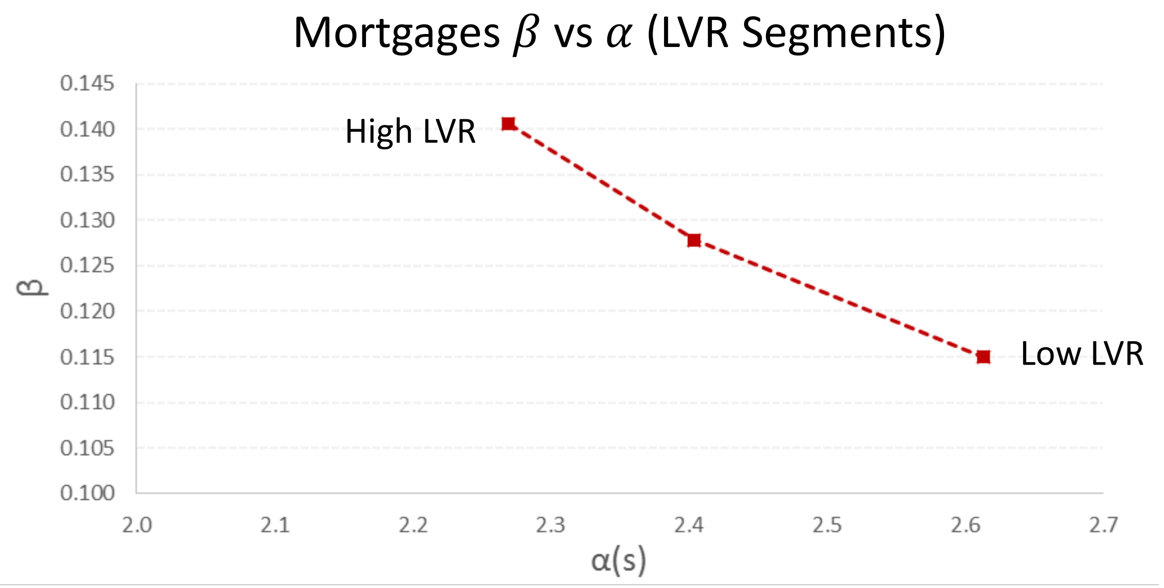
Loan to value ratio was tested independently as a factor influencing cycle sensitivity, using the same approach as the risk grades model. **Three LVR segments (high, medium, low)** were defined and all point in time PD model parameters were fit for each segment.

$$-\Phi^{-1}[PD_i] = \alpha_i + \beta_i \cdot Z + N\left(0, \sigma_i^2\right)$$

$$(\alpha, \beta, \sigma)(LVR) = \begin{cases} \left(\alpha'_1, \beta'_{1,} \sigma'_1\right) & if \quad LVR < l_1 \\ \left(\alpha'_2, \beta'_{2,} \sigma'_2\right) & if \quad l_1 \leq LVR < l_2 \\ \left(\alpha'_3, \beta'_{3,} \sigma'_3\right) & if \quad LVR \geq l_2 \end{cases}$$

Results – LVR Model

Whilst the LVR state of the cycle co-efficient shows significantly less variation than the risk grade model, the **higher LVR segment had higher state of the cycle coefficients than lower LVR segments.**



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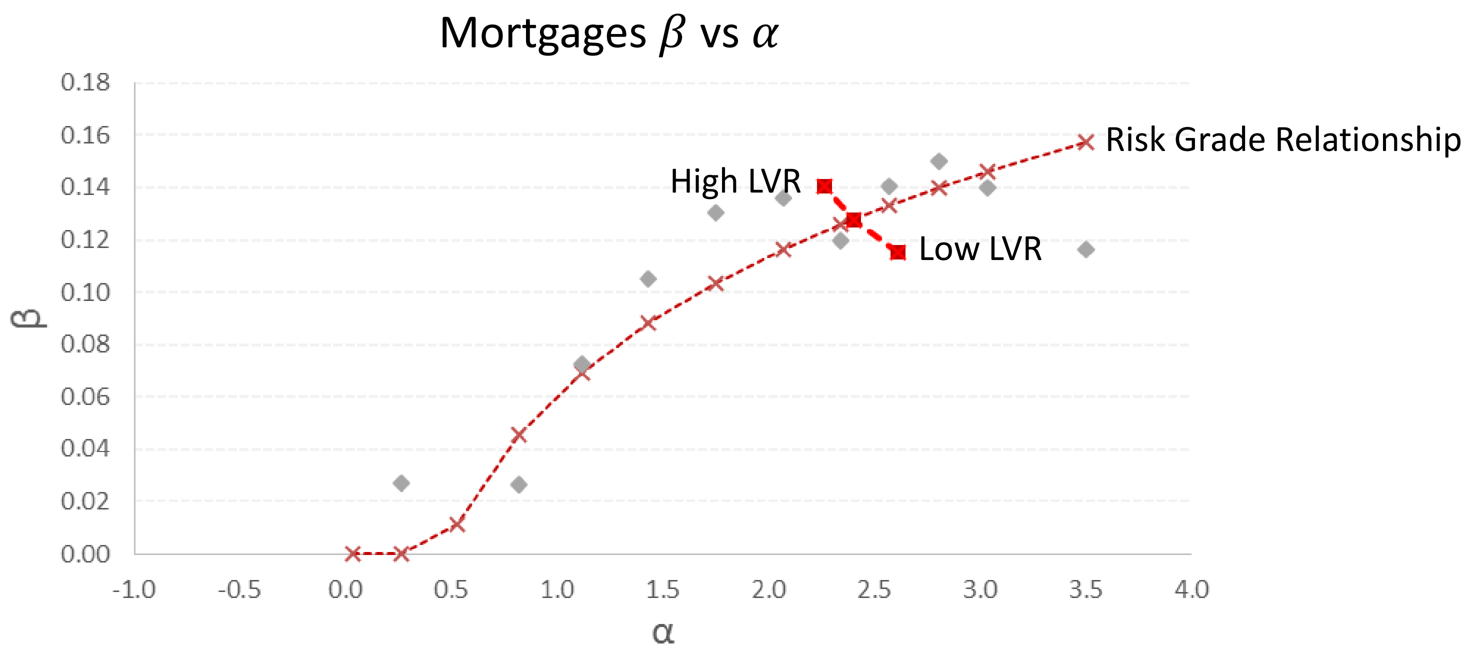
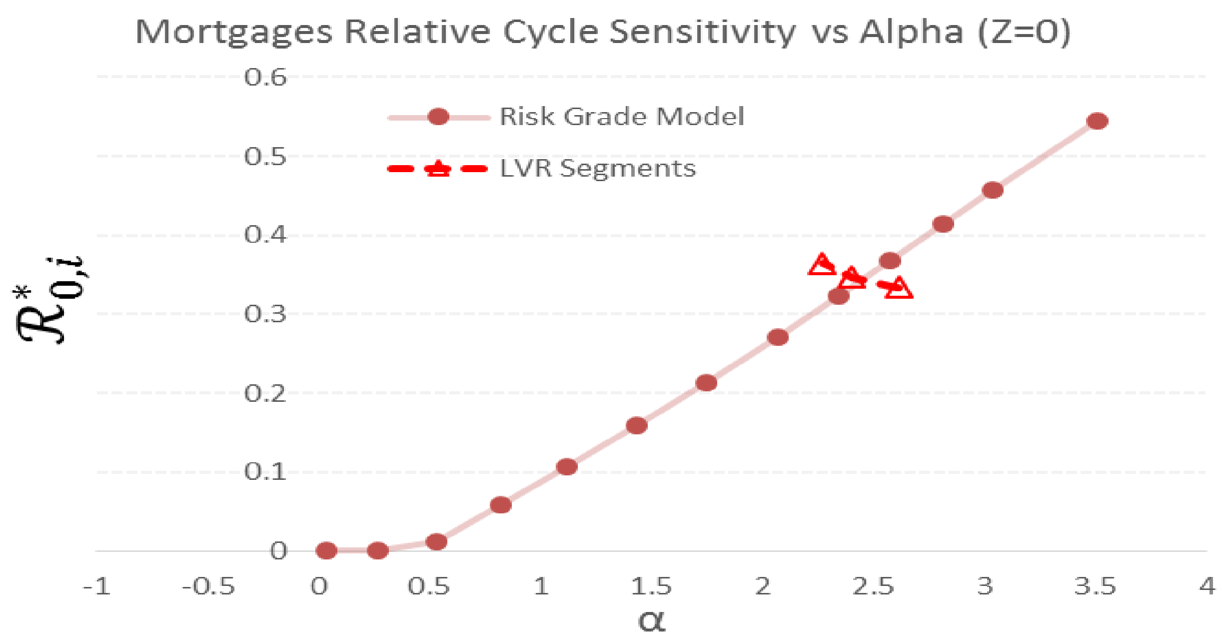


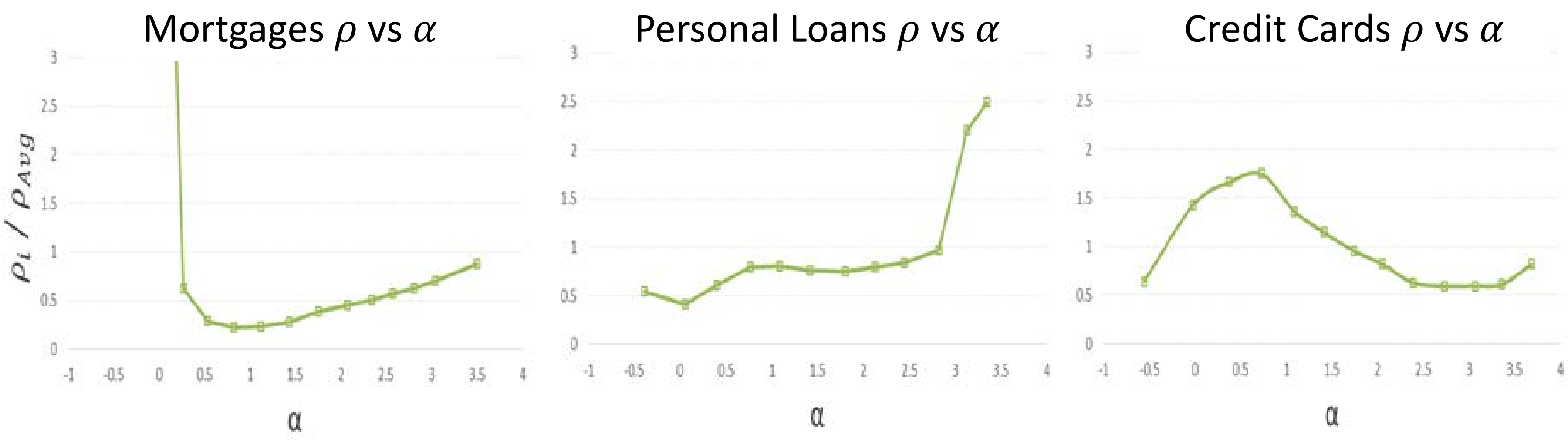
Results – LVR Model

The LVR effect is apparent over a much narrower range of credit quality (equivalently average PD) i.e. could be viewed as a **second order effect** (when compared to the credit quality effect).



Results – Asset Correlation Estimates

Asset correlation estimates show less intuitive outcomes. This is largely due to the Merton-ASRF model requiring some relatively unrealistic assumptions and (we would argue) asset correlations not being an easily interpreted or comparable measure of cycle sensitivity.



Note:

Y-axis on charts is the ratio of each grade asset correlation to grade average asset correlation (i.e. shows grade asset correlation relative to average asset correlation for the product).

Interpreting Results

Interpreting Results

Relative Cycle Sensitivity Relationships

- **Average PD effect** - A decreasing relationship between relative cycle sensitivity and average PD, as a general rule, makes sense given a very small cycle-driven increase in the number of defaults for a low PD segment can result in a large percentage increase in PD (**low denominator effect**).
- **Negative equity effect** - A higher relative cycle sensitivity for high LVR mortgages, all else being equal, also makes sense given greater likelihood of negative equity triggering a default for high LVR loans in a downturn.

Asset Correlation Relationships

- Asset correlation estimates not really a good measure of cycle sensitivity.

Conclusions

- By constructing precise measures of cycle sensitivity with respect to an observable state of the cycle indicator, we found some clear and intuitive trends.
- These trends are less evident for more traditional measures of systemic risk such as the asset correlation parameter within the popular Merton asymptotic single risk factor framework.
- Results have a number of practical applications for capital management, provisioning and underwriting practices.
- This is fairly easy to do, why not give it a try!

Thank You!

Addendum

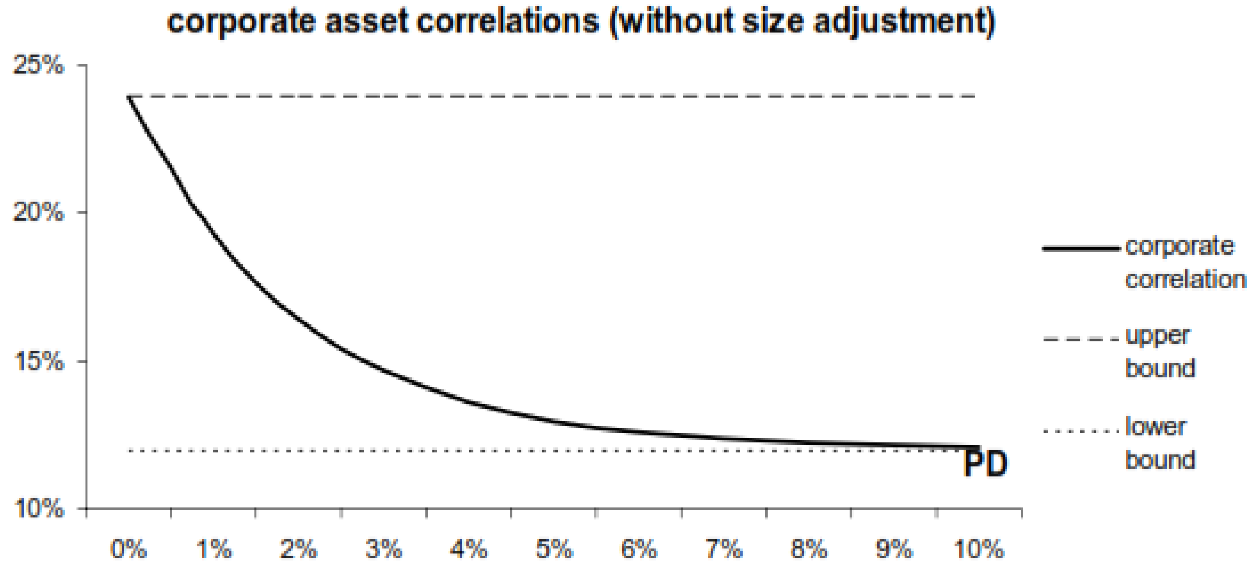
What have others found?

There are a number of studies related to **asset correlation dependencies** on a range of factors such as size, asset class and average probability of default.

Unfortunately there is no clear consensus as **many studies appear contradictory** in terms of both empirical findings and intuitive arguments for results, particularly regarding the **relationship between asset correlation and average probability of default**.

What have others found?

“ ***Asset correlations decrease with increasing PDs***. *This is based on both empirical evidence and intuition. Intuitively, for instance, the effect can be explained as follows: the* ***higher the PD, the higher the idiosyncratic (individual) risk components of a borrower.*** *The default risk depends less on the overall state of the economy and more on individual risk drivers ” [BIS, 2005]**



* *Reference:* An Explanatory Note on the Basel II IRB Risk Weight Functions, *BIS 2005*

What have others found?

*"For corporate exposures, **there is <u>no</u> strong decreasing relationship between average asset correlation and default probability** when firm size is properly accounted for.….*

*……..**sub prime borrowers are more sensitive to general economic conditions** and thus experience greater asset correlations than prime borrowers." [Moodys, 2009]**

Retail Asset Correlations – Prime vs Sub-Prime

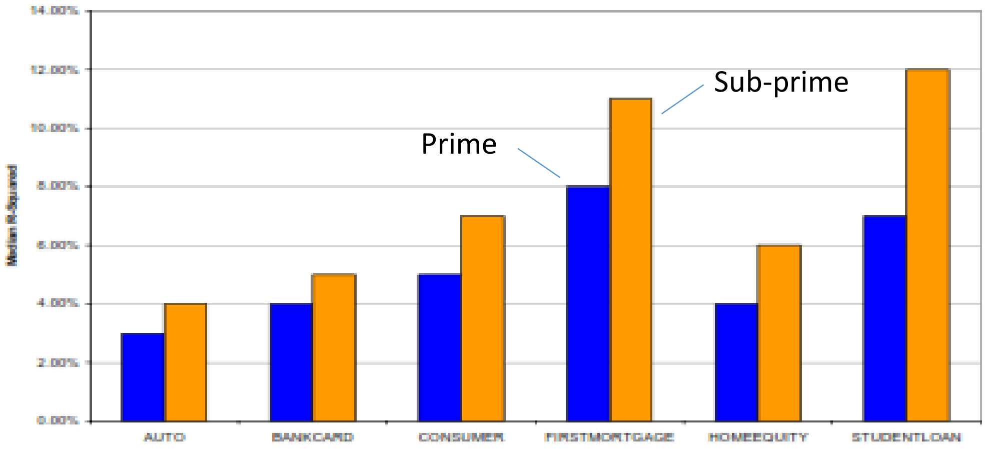


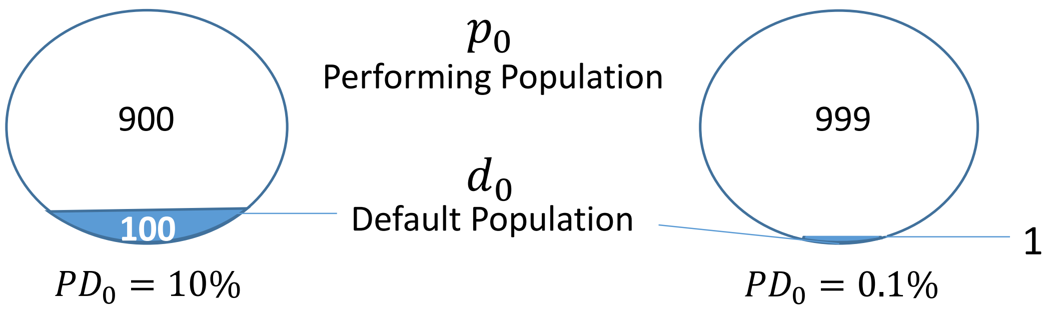
FIGURE 21 GCorr Retail Median R-squared Estimates

* *Reference:* The Relationship Between Average Asset Correlation and Default Probability, *Moodys, 2009*

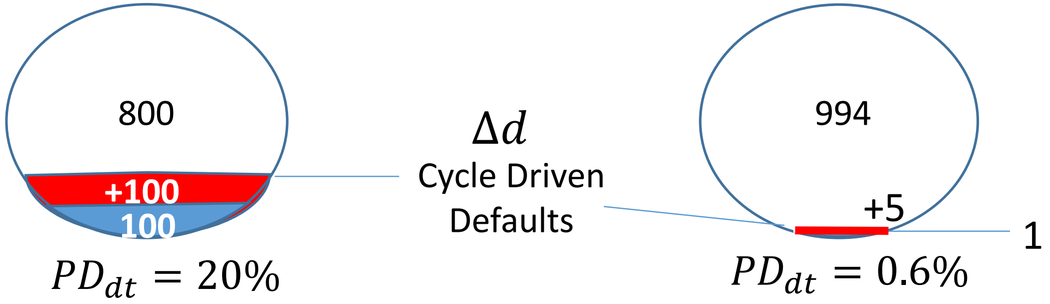
Interpreting Results – Credit Quality Relationship (Example)

Consider samples from a high risk and a low risk segment in benign economic conditions…

High Risk Segment: Low Risk Segment:



Economic downturn introduces **105** additional defaults due to job losses, 100 are absorbed by the high risk segment…



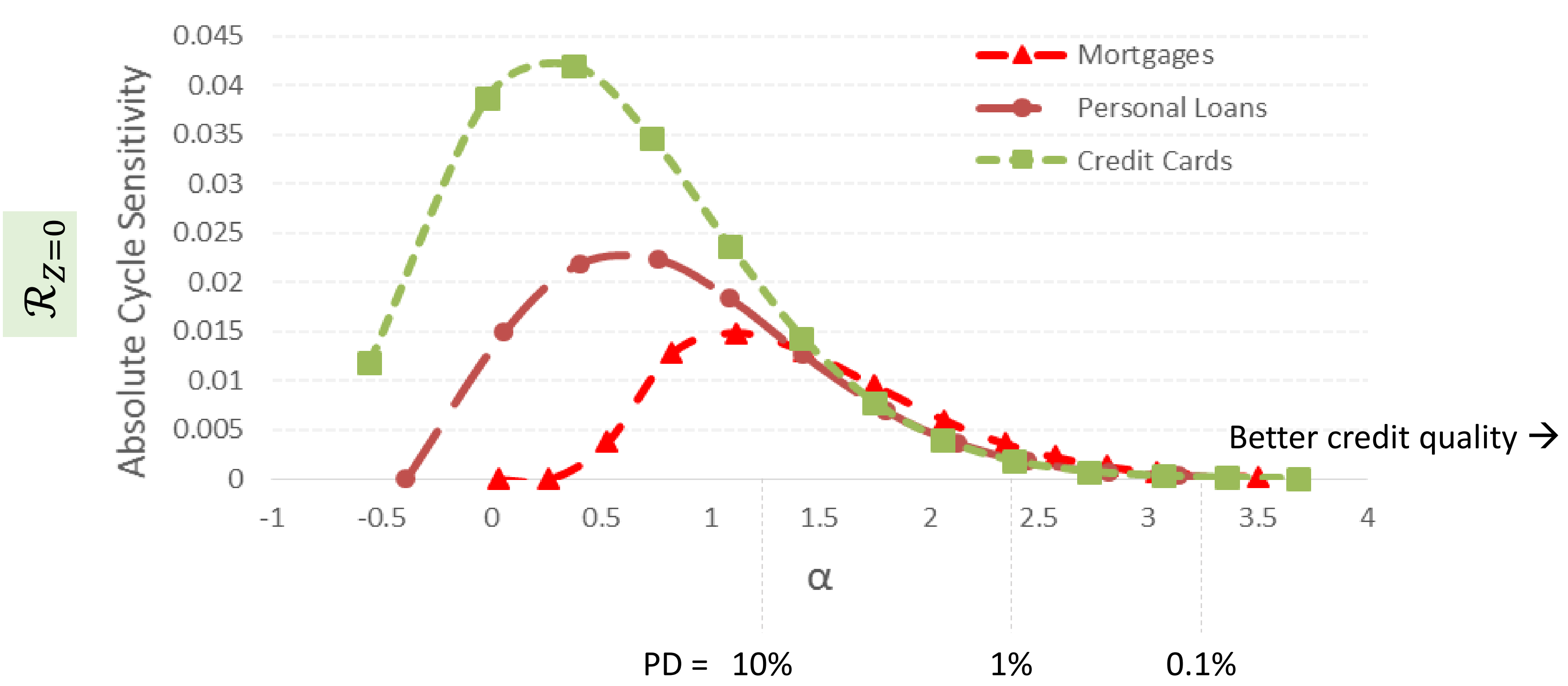
Two-fold increase in default rate.
Absorbs ~95% of cycle driven defaults

Six-fold increase in default rate – despite absorbing only 5% of cycle driven defaults.

Results – Absolute Cycle Sensitivity

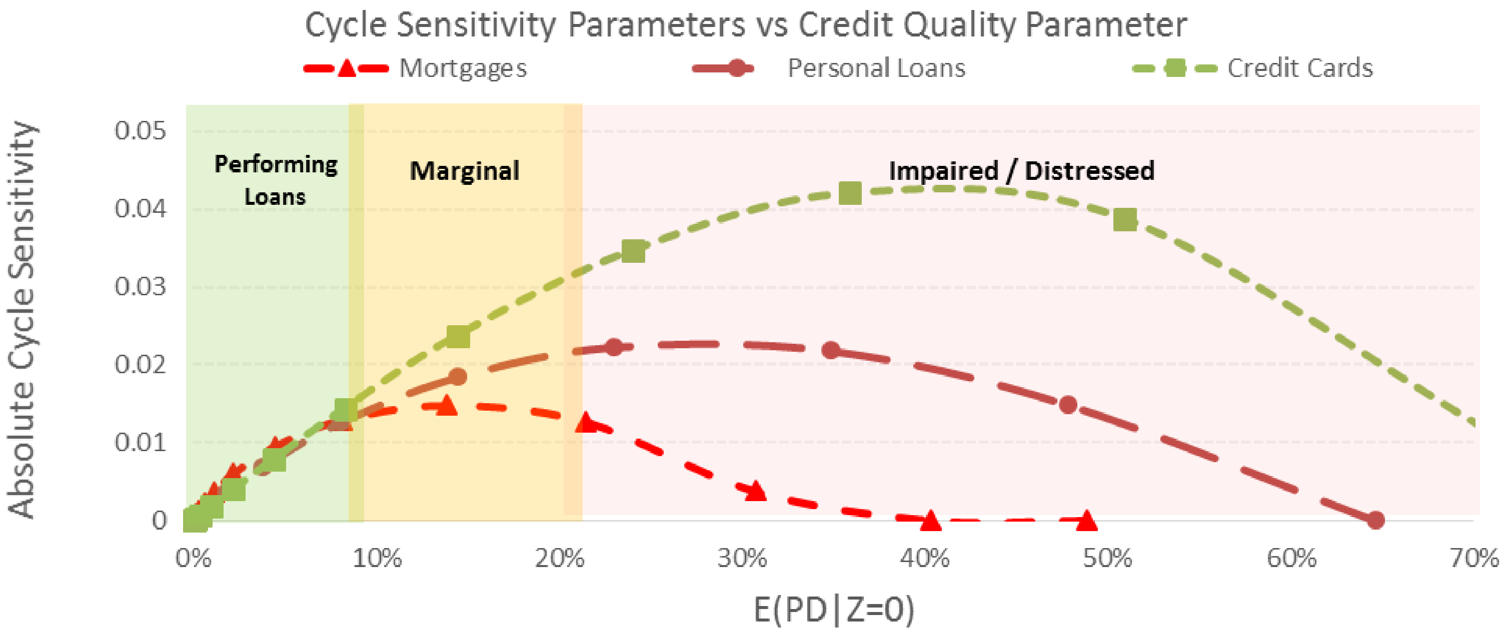
Absolute cycle sensitivity was found decrease with increasing credit quality for the majority of the rage of α, however reaches a **maximum at the 3-4th highest risk grades** before reducing to zero (or close to zero for the very highest risk grades.

Absolute Cycle Sensitivity vs α



Results – Cycle Sensitivity

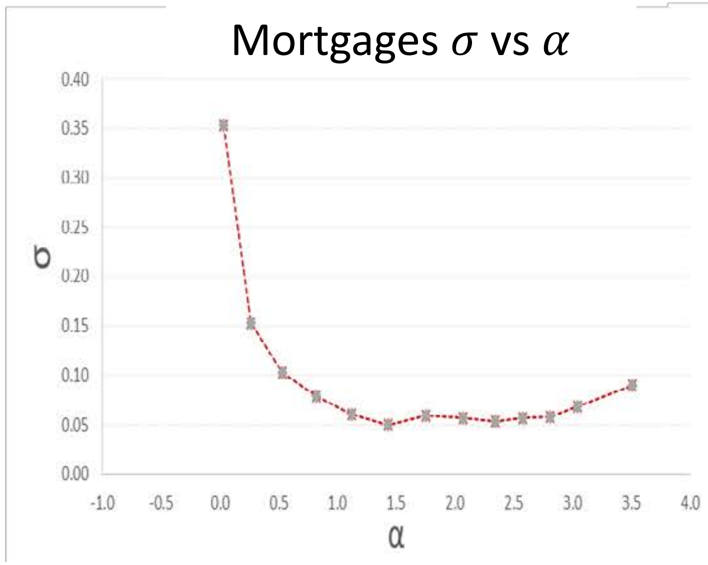
Portfolio cycle sensitivity was found to be inversely proportional to portfolio average PD…

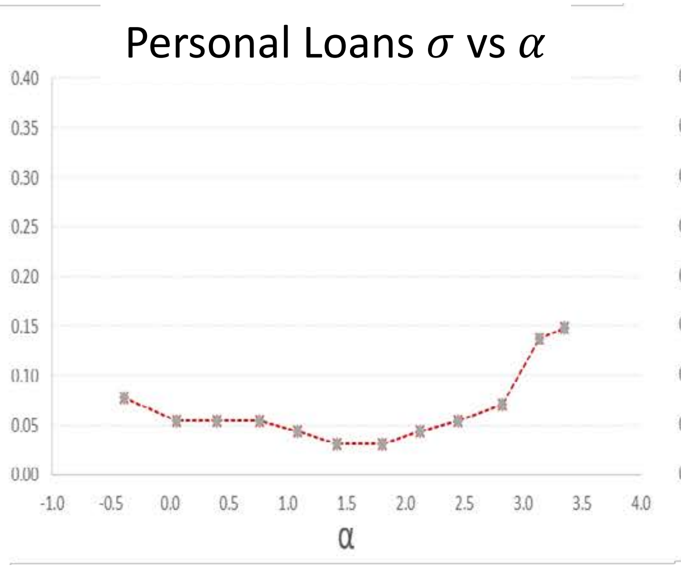


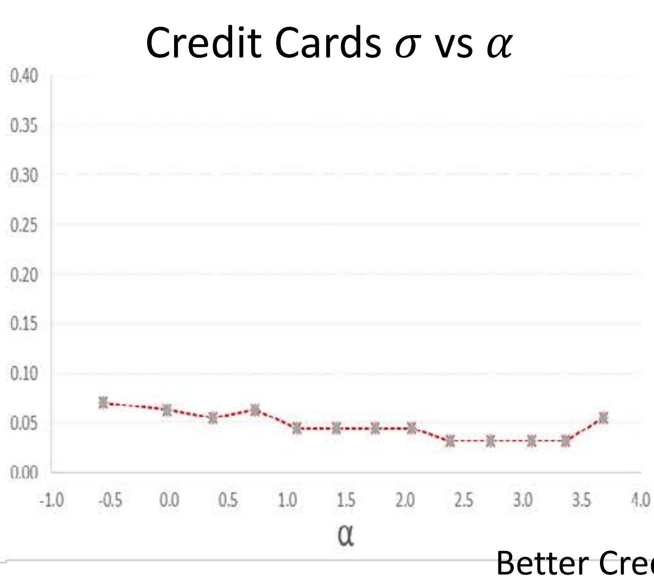
Results – Sigma

Residual volatility tended to be higher for the highest risk grades and in some cases the lowest risk grades (particularly for personal loans)

$$G^{-1}(PD) = \alpha + \beta \cdot Z + N(0, \boldsymbol{\sigma(\alpha, product)}^2\ )$$







Better Credit Quality →
(lower PD)

Interpreting Results – Asset Correlation

By developing a 2-factor model we were able to **separate default rate dependency on an observable state of the cycle indicator from residual volatility not linked to the economic cycle**. Given our results, we would suggest estimating asset correlations under a Merton-ASRF model using portfolio default data inevitably **blends cycle driven and non-cycle driven effects.**

$$\rho = \frac{\beta^2 + \sigma^2}{1 + \beta^2 + \sigma^2} \approx \beta^2 + \sigma^2 \quad if \quad \beta^2 + \sigma^2 \ll 1$$