Markov chains in the dynamic control of credit consumer portfolios

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Credit Scoring and Credit Control XI

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Lorenzo Quirini
Consum.it – Monte dei Paschi di Siena Group
Head of "Servizio Sistemi Decisionali, Monitoraggio e Scoring"

Luigi Vannucci
University of Florence

Objective of the presentation

We present a methodology for measuring dynamically the creditworthiness for retail portfolios in situations of significant change in economic context

- **The tools to be used**
  - **Creditworthiness Index (CWI)**
  - **Markov chains (both homogeneuos or not)**
  - **Survival analysis with competing risks**

- **Their uses**
  - **Underwriting**
  - **Collection**
  - **Profit/Loss estimation**

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The agenda

- CONSUM.IT
- THE MACROECONOMIC CONTEXT AND NEW RISKS TO BE FACED
- THE TOOLBOX TO FACE SUCH NEW RISKS

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Consum.it as part of MPS Group

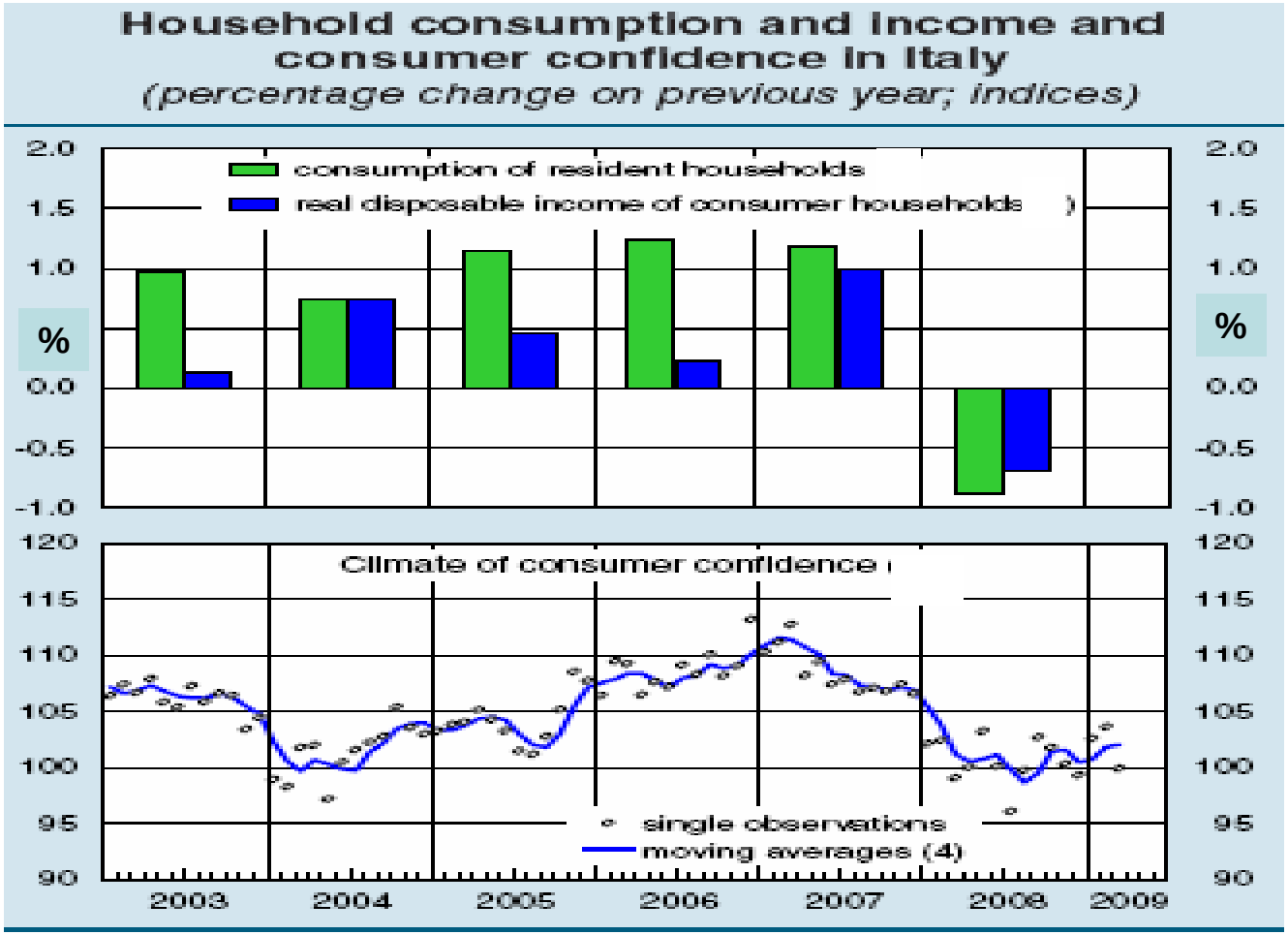
- **Monte dei Paschi di Siena was founded in 1472 and is the oldest Bank in the world! Nowadays, MPS is the 3rd Banking Group in Italy with more than 3000 branches and ~5 million of customers**

- **Consum.it is a "stand alone" company fully owned by MPS group, being its consumer finance company**

- **Consum.it was founded in 1998 and now offers a complete range of consumer finance products: its business started with purpose loans (in 1999), then it launched revolving card (in 2002), and personal loans (in 2004)**

Consumptions and household incomes have declined and this trend has reduced the consumer confidence

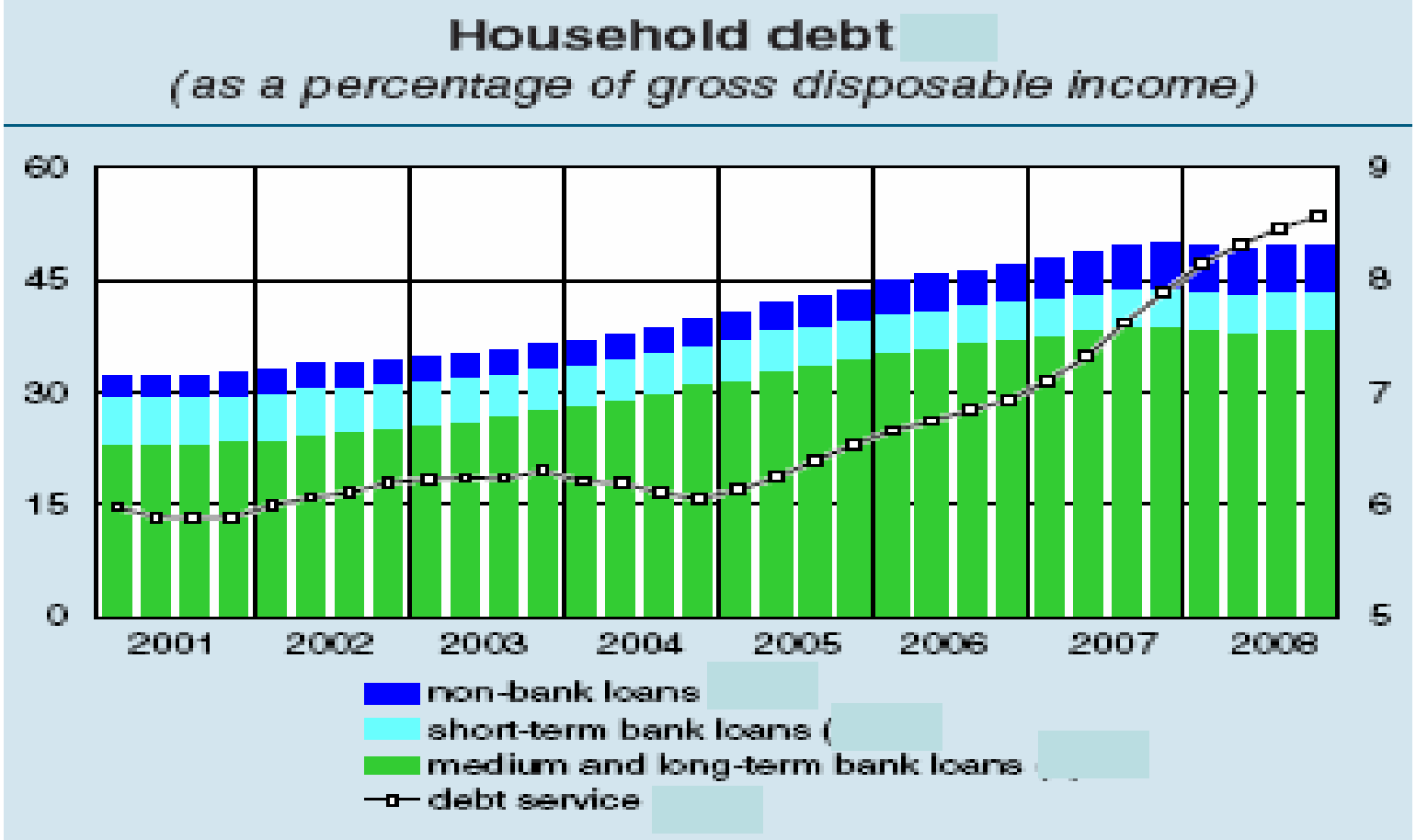
Consumptions, household incomes and confidence



Source : Economic Bullettin n.56, April 2009 – BANK OF ITALY

Household debt has decreased slightly; debt service has reached its maximum since 2001

Household debt and debt service



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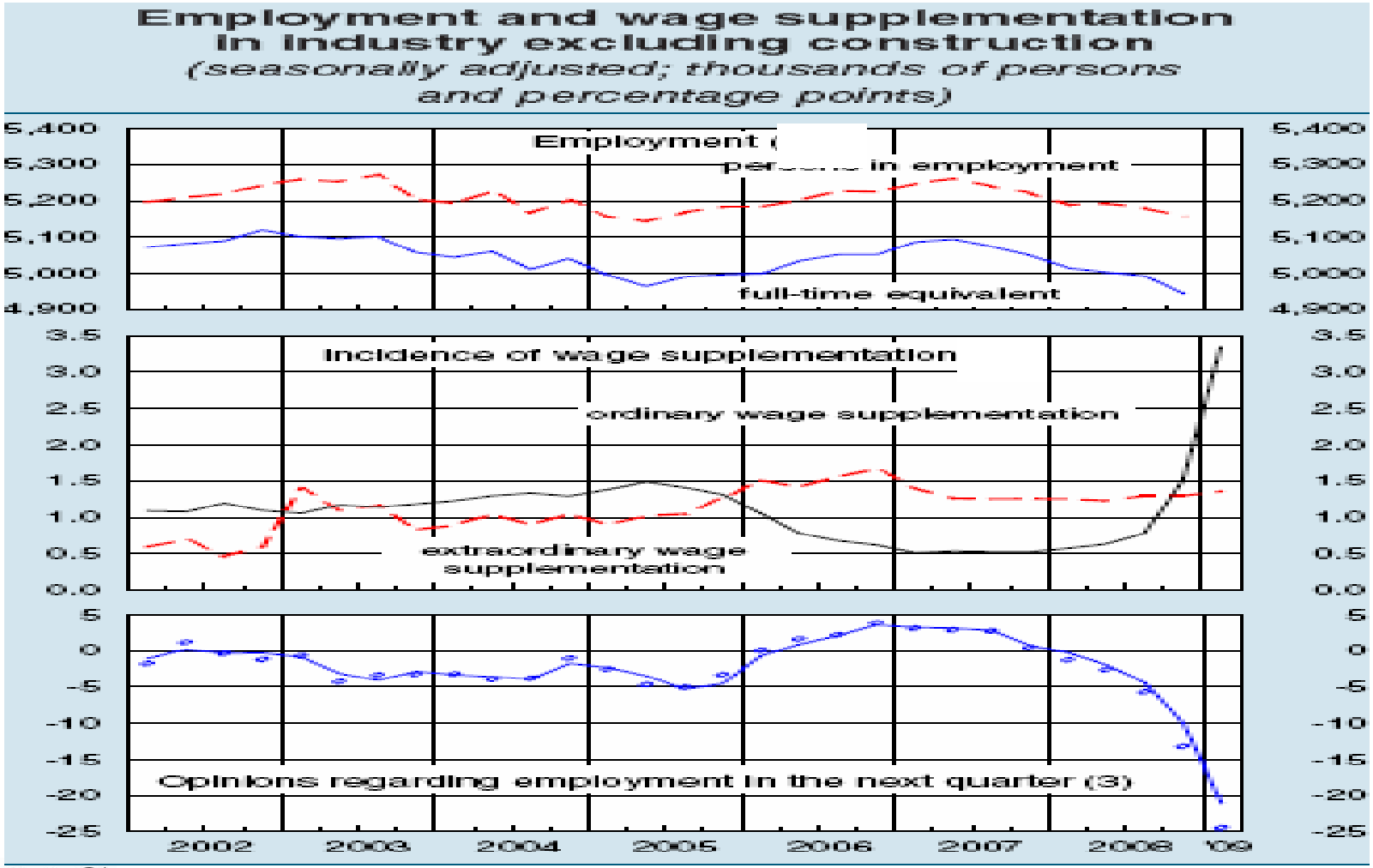
Employment has fallen, it has intensified wage supplementation, the expectations about employment are collapsed

The level and employment expectations



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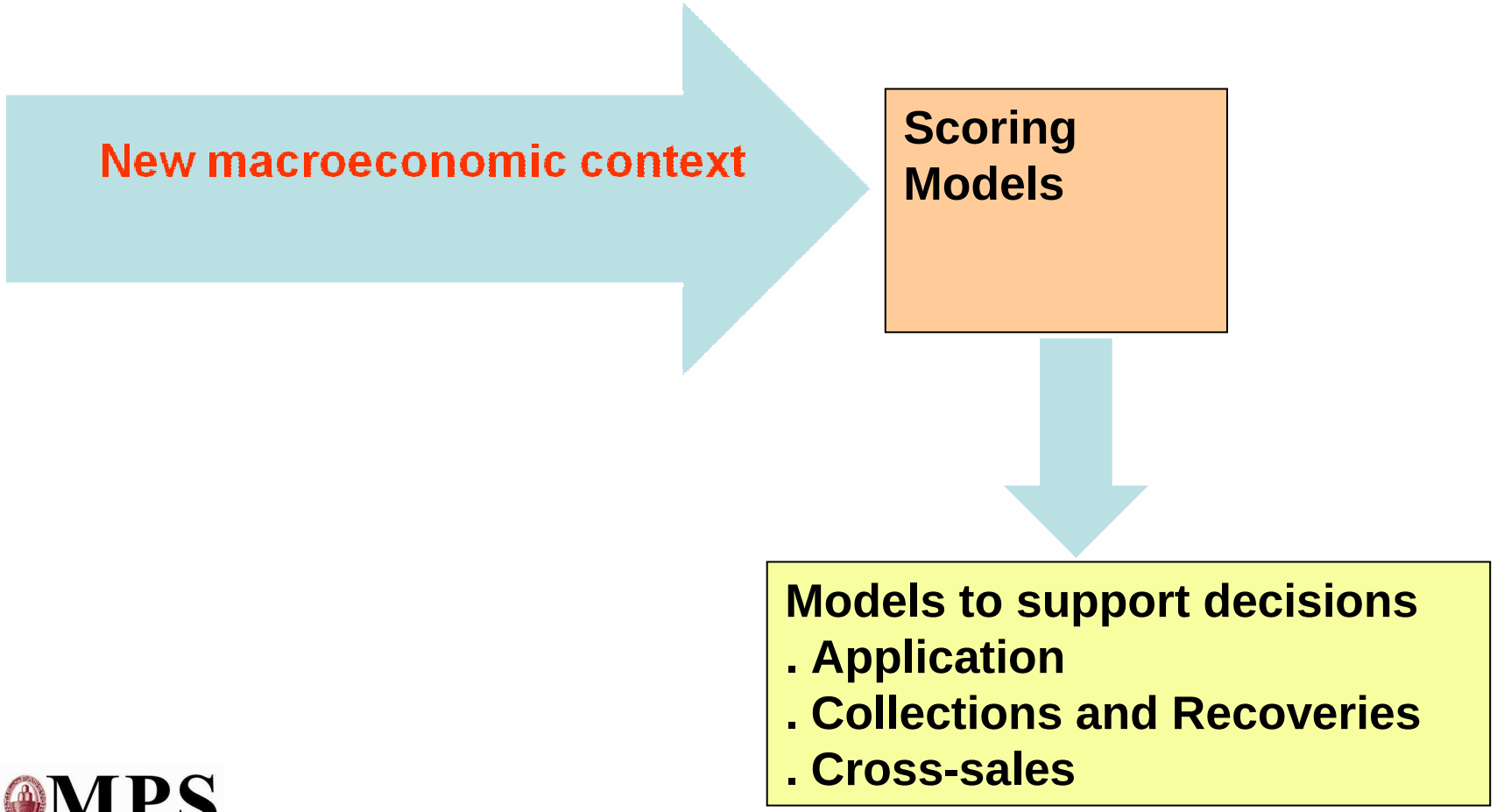
The priority is the selection of analytical tools to measure quickly the change in credit quality

The new risks to be faced



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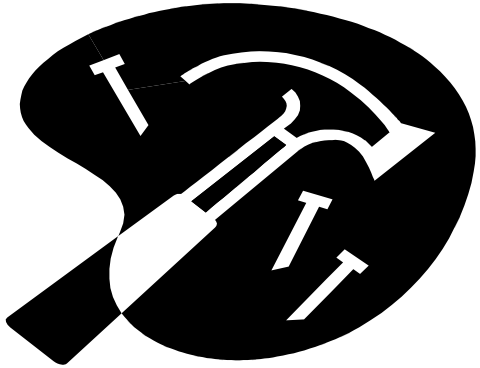
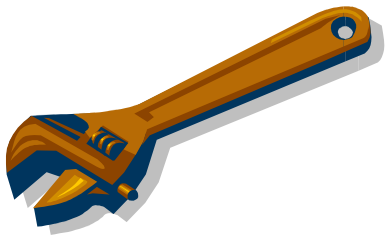
Several tools have to be managed

The toolbox



CREDITWORTHINESS INDEX (CWI): HOW CUSTOMERS FULFILL THEIR FINANCIAL OBLIGATIONS



MARKOV CHAINS: HOW TO FOLLOW A CREDIT PORTFOLIO OVER TIME

SURVIVAL ANALYSIS WITH COMPETING RISKS: NOT “IF” BUT “WHEN”

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The common element of these tools is their dynamic character

Two possible point of view: static or dynamic

Static approach: the estimate of the response variable over a fixed time horizon

- Example: default probability over the next 12 months

Dynamic approach: the estimate of the response variable for each observation time

- Example: survival probability (to default) at time 1, 2, 3,…, 12

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CWI: the definition (fixed term loan)

$$X_t = \frac{\sum_{h=1}^{t} R_h (1+i)^{-h}}{\sum_{h=1}^{t} r_h (1+i)^{-h}}$$

R_h installment (random) given at time *h*

***h*= 1, …, *m* (*m* term of the operation)**

***t* evaluation time (1≤*t*≤*m*)**

r_h contractual installment at time *h*

***i* internal rate of return**

Case 1

If $R_h = a r_h \ (a \le 1)$ then

$$X_t = \frac{\sum_{h=1}^{t} R_h (1+i)^{-h}}{\sum_{h=1}^{t} r_h (1+i)^{-h}} = \ldots = a$$

Case 2

p = prob. payment each installment

$$E(X_t) = E\left(\frac{\sum_{h=1}^{t} R_h (1+i)^{-h}}{\sum_{h=1}^{t} r_h (1+i)^{-h}}\right) = \ldots = p$$

Case3

j = number delayed months

$$X_{m+j} = \frac{\sum_{h=1}^{m} r_h (1+i)^{-(h+j)}}{\sum_{h=1}^{m} r_h (1+i)^{-h}} = (1+i)^{-j}$$

Quirini, L., Vannucci, L., 2009. "A new index of creditworthiness for retail products", in *Journal of the Operational Research Society,* doi:10.1057/jors.2009.68.

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The relationship between CWI and internal rate of return (Irr)

$$B = \sum_{h=1}^{m} r_h (1 + x)^{-h}$$

B **granted amount**

m **term of the operation**

r_h **installment due at time** ***h*** **(*h* = 1, …, *m*)**

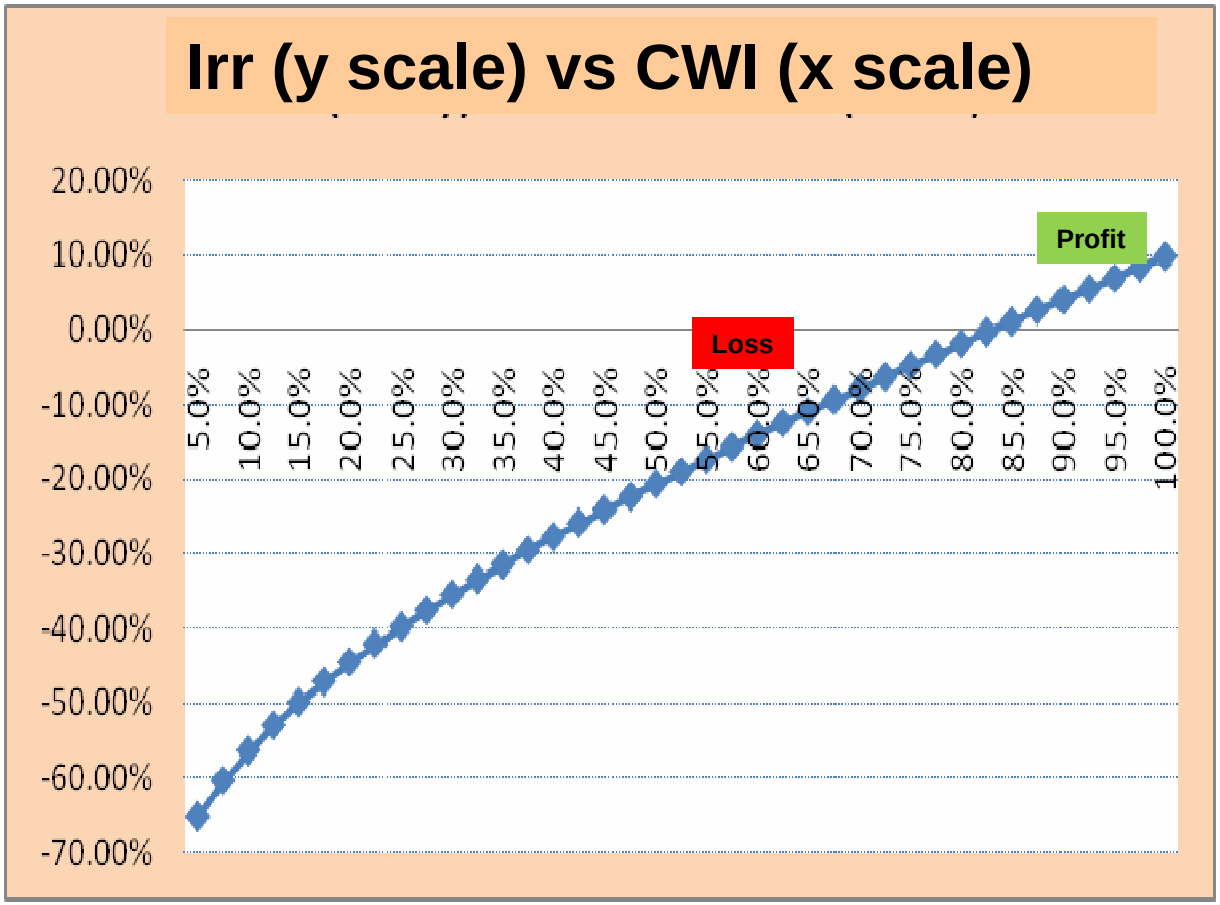
x **is the contractual rate of return**

$$B = \sum_{h=1}^{m} a \cdot r_h (1 + y)^{-h}$$

a **CWI (0≤ *a* ≤1)- case 1**

y **is the internal rate of return**

Loan B=12000 euro, m = 48 months, APR = 10.0%



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Markov chains

Class of random processes that describes the evolution over time of a "system" which can take a finite number of “states "

The system:

- **a set of customers with the same creditworthiness (similar application scores)**
- **a set of customers with the same entry in collections**

The states:

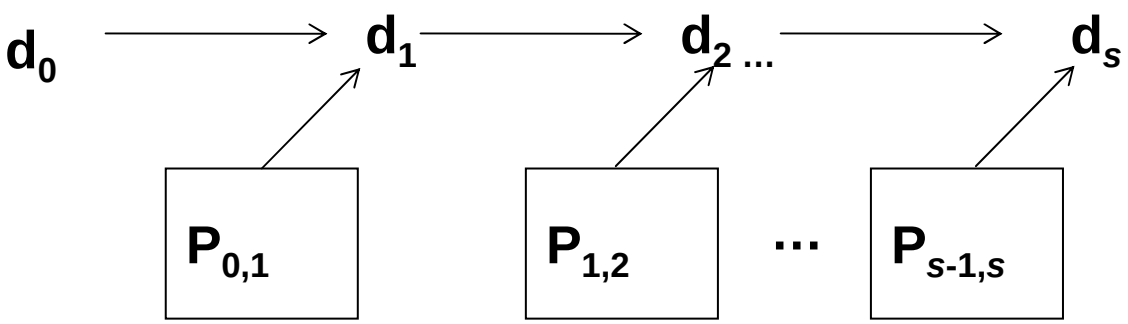
- **a set of customers with similar CWI**
- **a set of customers with the same classification in collections**

We have followed the population over *s* months

The model

$$\mathbf{d}_h = \mathbf{d}_{h-1} * \mathbf{P}_{h-1,h}$$

$\mathbf{d}_h$ is the marginal distribution at time h ($R^{1,q}$)
$\mathbf{P}_{h-1,h}$ is the transition matrix from time h-1 to time h ($R^{q,q}$)
q is the number of states of the chain
$h = 1, 2, \ldots, s$



Marginal distributions

Non homogeneous transition matrices

The hystory from time 0 to time *s* has been used to estimate an homogenous matrix to make forecasts after time *s*

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Two different parameters have been estimated

The maximum likelihood estimates for transition probabilities (e.g, in a group with similar creditworthiness)

$$\hat{p}_{ij}(t) = \frac{n_{ij}(t)}{\sum_{j=1}^{q} n_{ij}(t)}$$

Case 1*:* non-homogeneous Markov chain

n_{ij} (t) denotes the number of customers in state i at t-1 and j at t
q is the number of states
t goes from 1 to s

$$\hat{p}_{ij} = \frac{\sum_{t=1}^{s} n_{ij}(t)}{\sum_{k=1}^{q}\sum_{t=1}^{s} n_{ik}(t)}$$

Case *2:* homogeneous Markov chain

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The link between CWI and Markov chain

$$E(CWI_c) = \sum_{k=1}^{q} w_k \cdot E(CWI_k)$$

w_k represents the probability that the customer belongs to class *k*

w_k have been estimated by Markov chain while CWI expectations and variances have been evaluated by historical data

$$\mathrm{var}(CWI_c) = \sum_{k=1}^{q} w_k \cdot E(CWI_k^2) - E^2(CWI_c)$$

CWI_c stands for the Creditworthiness index for a customer in a given subportfolio (similar creditworthiness), CWI_k stands for the class *k*

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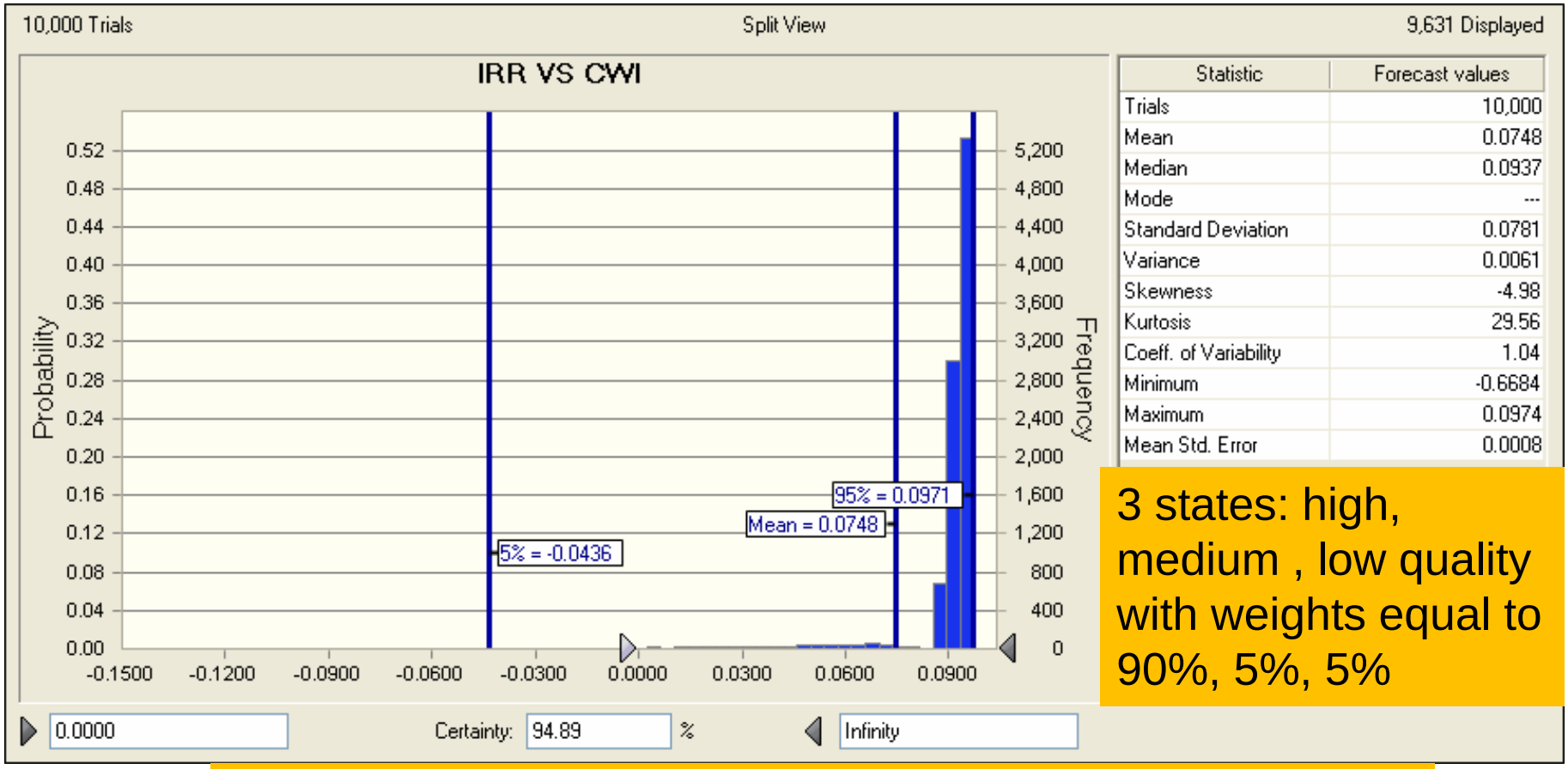
An example based on simulation: how to evaluate IRR distribution

Simulation with 10000 trials; granted amount = 12000 euro, term = 48 months; APR=9.74%, for a “mixture” customer in a given sub-portfolio



Statistic	Forecast values
Trials	10,000
Mean	0.0748
Median	0.0937
Mode	---
Standard Deviation	0.0781
Variance	0.0061
Skewness	-4.98
Kurtosis	29.56
Coeff. of Variability	1.04
Minimum	-0.6684
Maximum	0.0974
Mean Std. Error	0.0008

3 states: high, medium , low quality with weights equal to 90%, 5%, 5%

Triangular distributions (min, max, mode) for CWI in the states: High (0.98, 1, 0.99), Medium (0.80,0.98, 0.95), Low (0, 0.80, 0.60)

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Survival analysis: the basic idea

The random time is the minimum between the following events:

- **Default (write-off)**
- **Censoring (time of last observation for subjects still at risk)**

An useful generalization
Competing risks for events which have a negative impact on the company’s profit & loss account:

- **Default (write-off)**
- **pre-payment (churn)**

The link between CWI, Markov chains and survival analysis

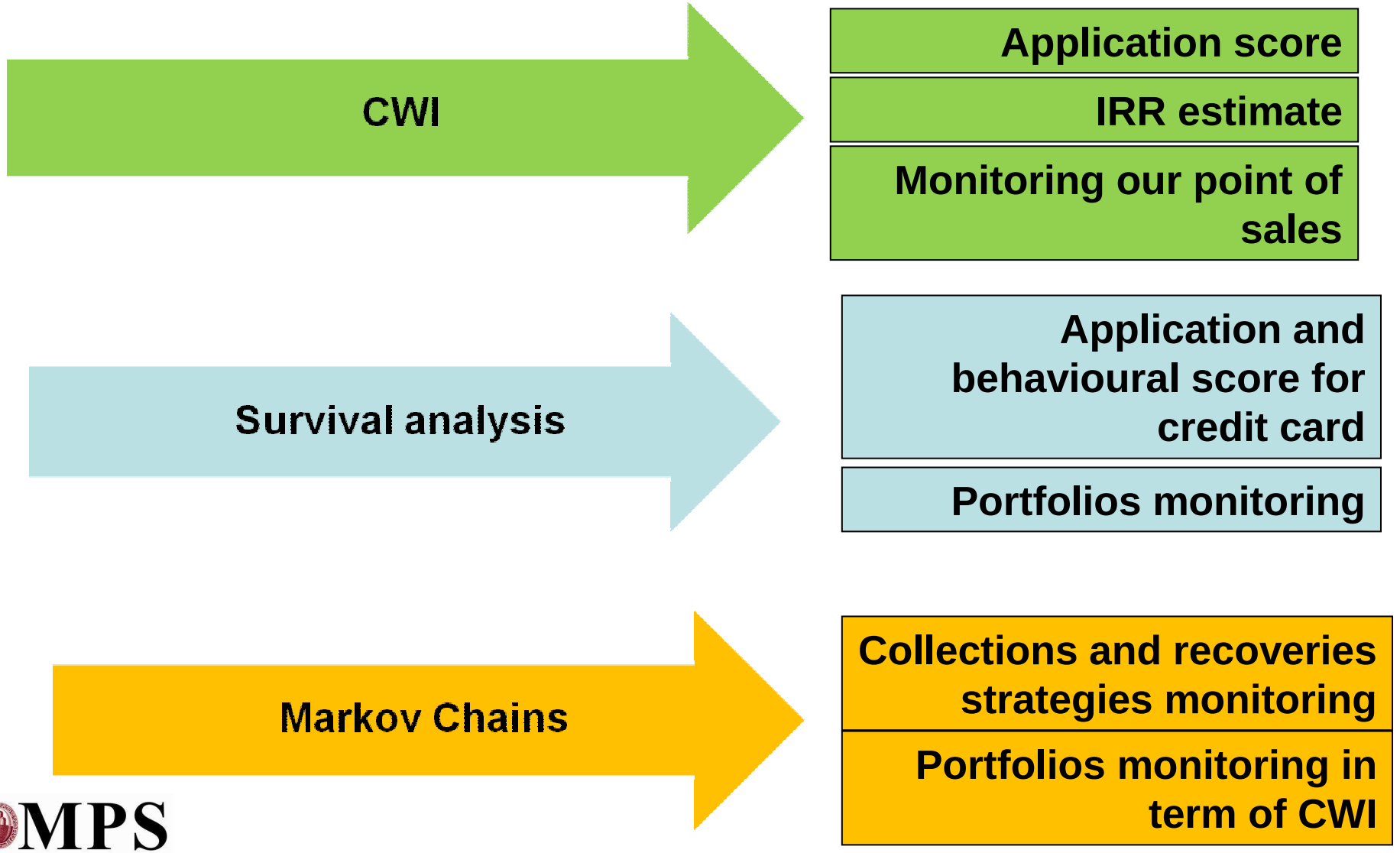
- CWI values have been used to define states in Markov chains approach

- The possible outcomes given in competing risks (write-off, churn) may be used as “absorbing” states in Markov chains

- CWI has a direct interpretation in terms of PD, LGD, EAD

How we have used such tools

The applications



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