BEAUTIFUL INTERLINKS IN CREDIT SCORING

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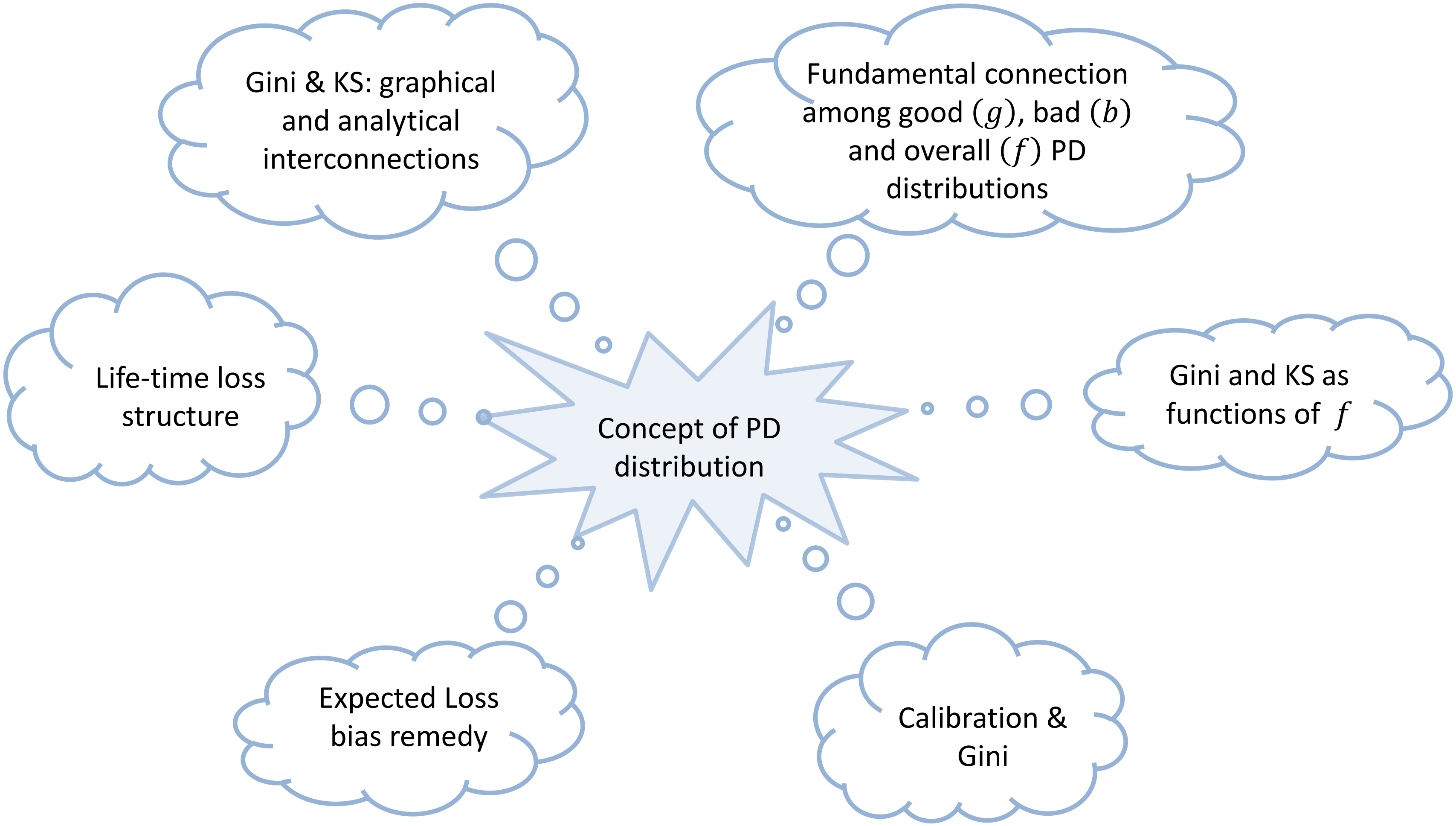
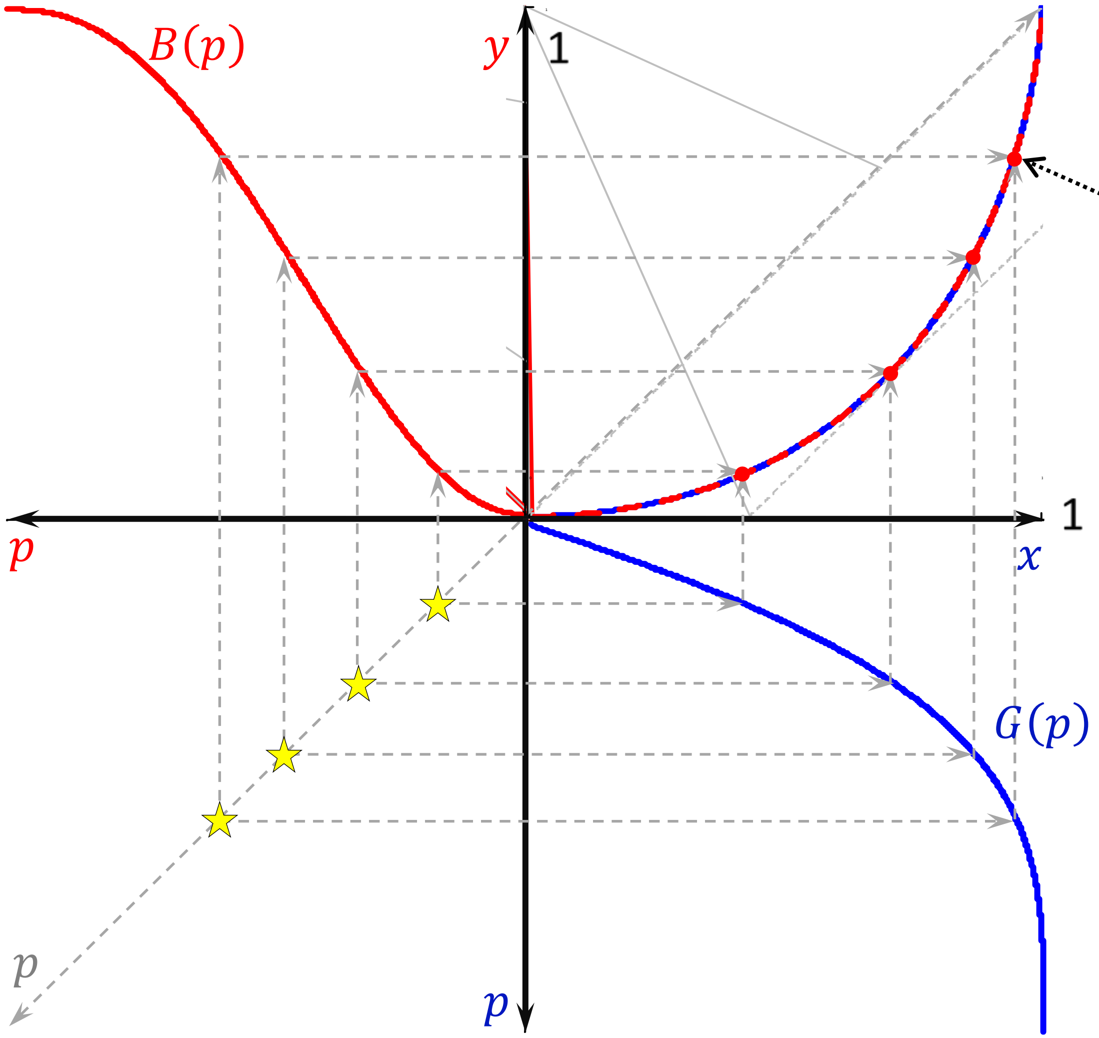
2017

Agenda


Gini & KS: graphical and analytical interconnections
Fundamental connection among good (g), bad (b) and overall (f) PD distributions
Life-time loss structure
Concept of PD distribution
Gini and KS as functions of f
Expected Loss bias remedy
Calibration & Gini

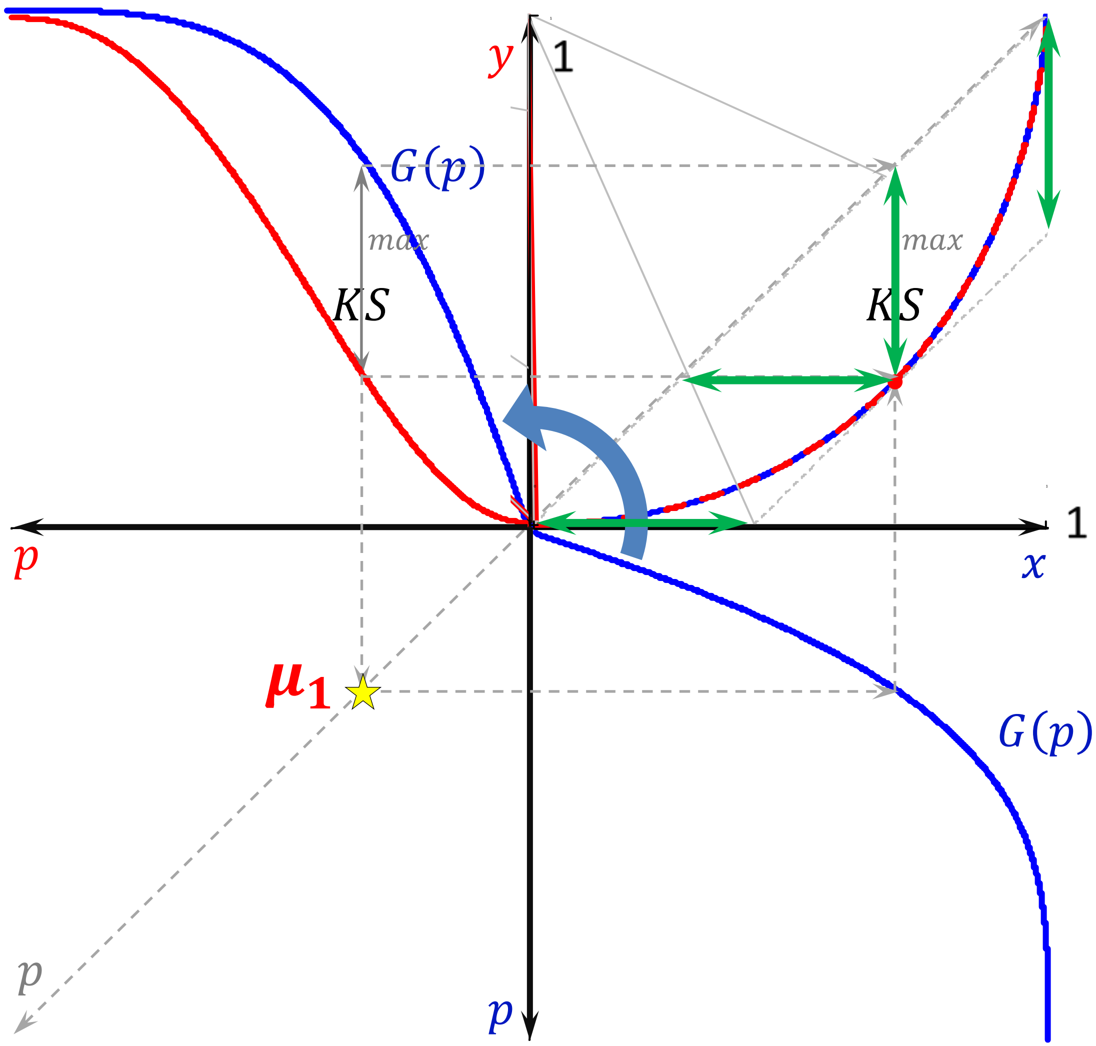
Gini & KS: graphical interconnection

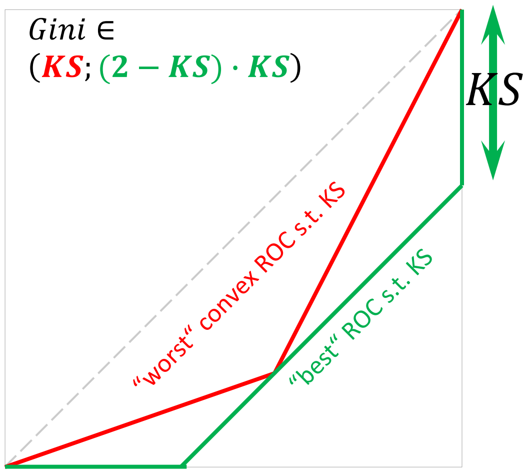


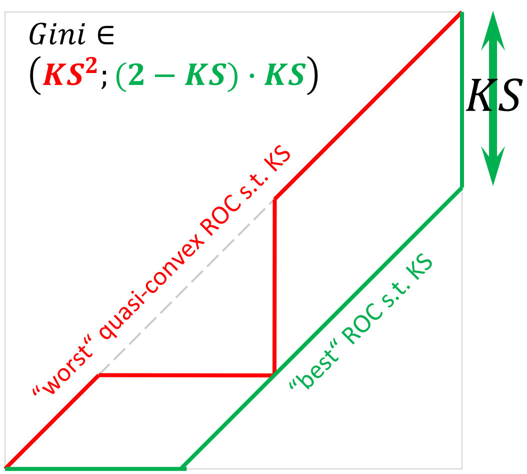
ROC curve defined parametrically:

$$\begin{cases} x = G(p) \\ y = B(p) \end{cases}$$

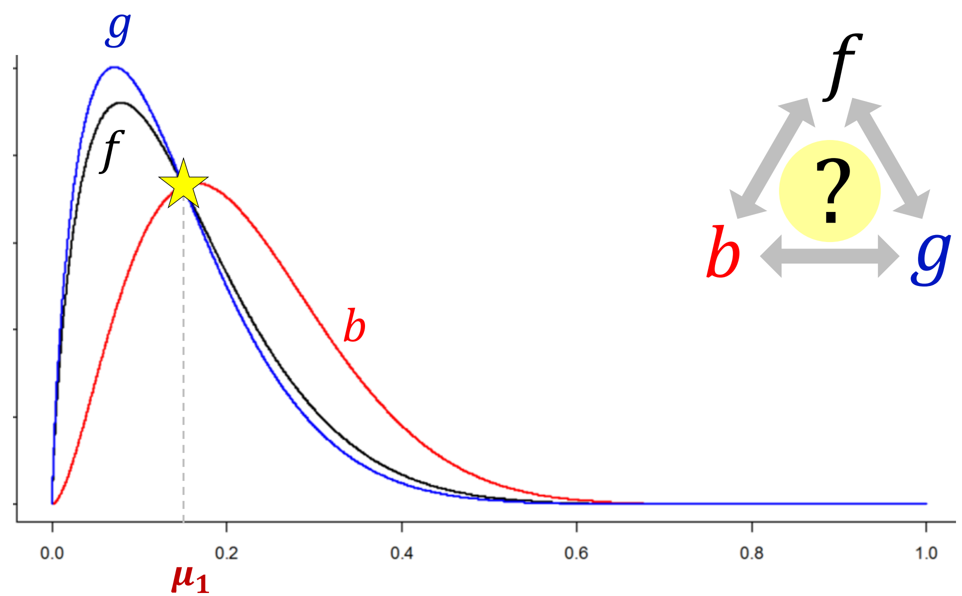
Gini & KS: graphical interconnection







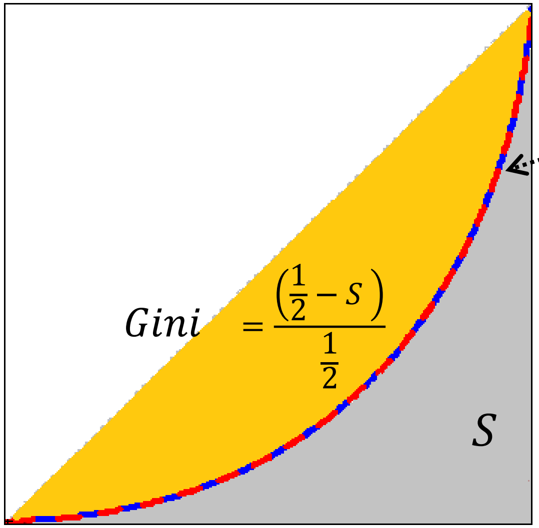
Fundamental connection among good (g), bad (b) and overall (f) PD distributions



$$b(p) = f(p|D) = \frac{f(D|\,p)\cdot f(p)}{f(D)} = \frac{p\cdot f(p)}{\mu_1} \qquad g(p) = \frac{(1-p)\cdot f(p)}{1-\mu_1}$$

$$f = \mu_1 \cdot b + (1-\mu_1)\cdot g$$

Gini and KS as functions of f



$$\begin{cases} x = G(p) \\ y = B(p) \end{cases}$$

$$S = \int_0^1 B(p)\, dG(p)$$

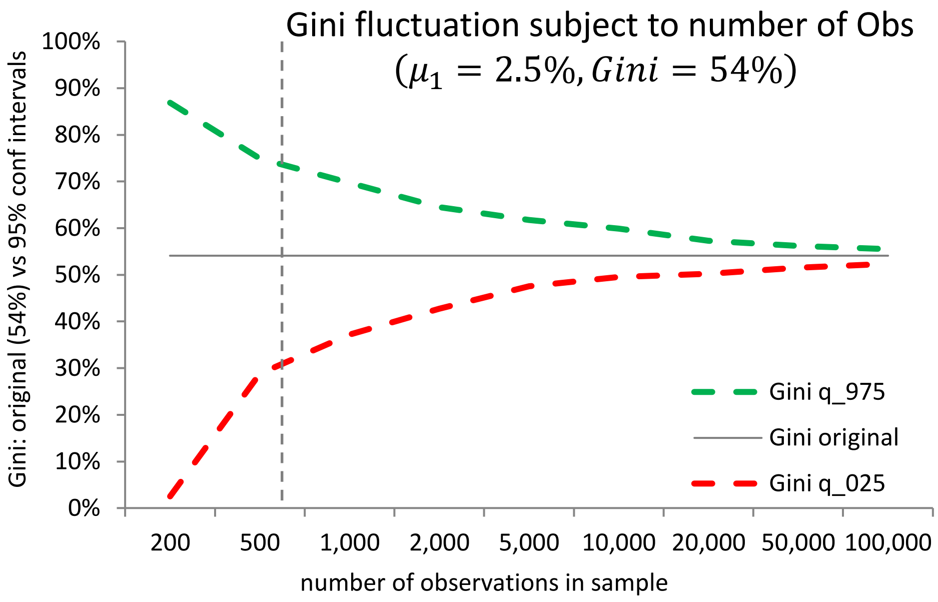
$$B(p) = \int \frac{pf}{\mu_1} dp$$

$$G(p) = \int \frac{(1-p)f}{1-\mu_1} dp$$

$$\Rightarrow \boxed{Gini = \frac{2 \cdot \int_0^1 p \cdot f \cdot F dp - \mu_1}{\mu_1(1-\mu_1)}}$$

$$\Rightarrow \boxed{KS = \frac{1}{\mu_1(1-\mu_1)} \cdot \int_0^{\mu_1} f \cdot (\mu_1 - p)\, dp}$$

NB: if $p \sim Beta(\alpha, \beta)$, then $Gini = -0.7125 \cdot KS^2 + 1.7881 \cdot KS - 0.0594 \quad (R^2 = 0.9999)$



1. To form a belief about μ_1 and intrinsic model $Gini$ (e.g., = $Gini$ from development sample)

2. From $(\mu_1, Gini)$ we uniquely derive parameters of corresponding Beta distribution (here assume: $p \sim Beta(\alpha, \beta)$)

3. To draw from defined Beta distribution a sample of PD’s of size n (i.e., size of our working validation sample)

4. To simulate outcomes (0,1) from sampled PD’s

5. To estimate Gini (as we have vector of PD’s and vector of realized outcomes)

6. To repeat 2-4 many times and to estimate Gini distribution and its respective percentiles

Max theoretical values of Gini

maximize $\left(Gini = \frac{2 \cdot \int_0^1 p \cdot f \cdot F dp - \mu_1}{\mu_1(1-\mu_1)} \right)$

subject to: $\begin{cases} p \sim Beta(\alpha, \beta) \\ f \text{ is unimodal} \\ \text{average PD} = \mu_1 \end{cases}$

$\Rightarrow$ solution: $\begin{cases} \alpha^* = \frac{\mu_1}{1-\mu_1} \\ \beta^* = 1 \end{cases}$

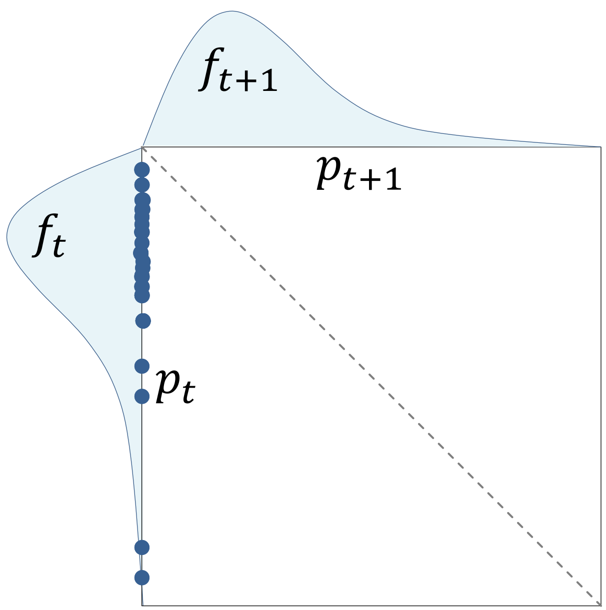
For example,
for $\mu_1 = 5\%$, max theoretical Gini is 95%;
for $\mu_1 = 20\%$, max Gini = 83%;
for $\mu_1 = 50\%$, max Gini = 67%; …

Life-time loss estimation

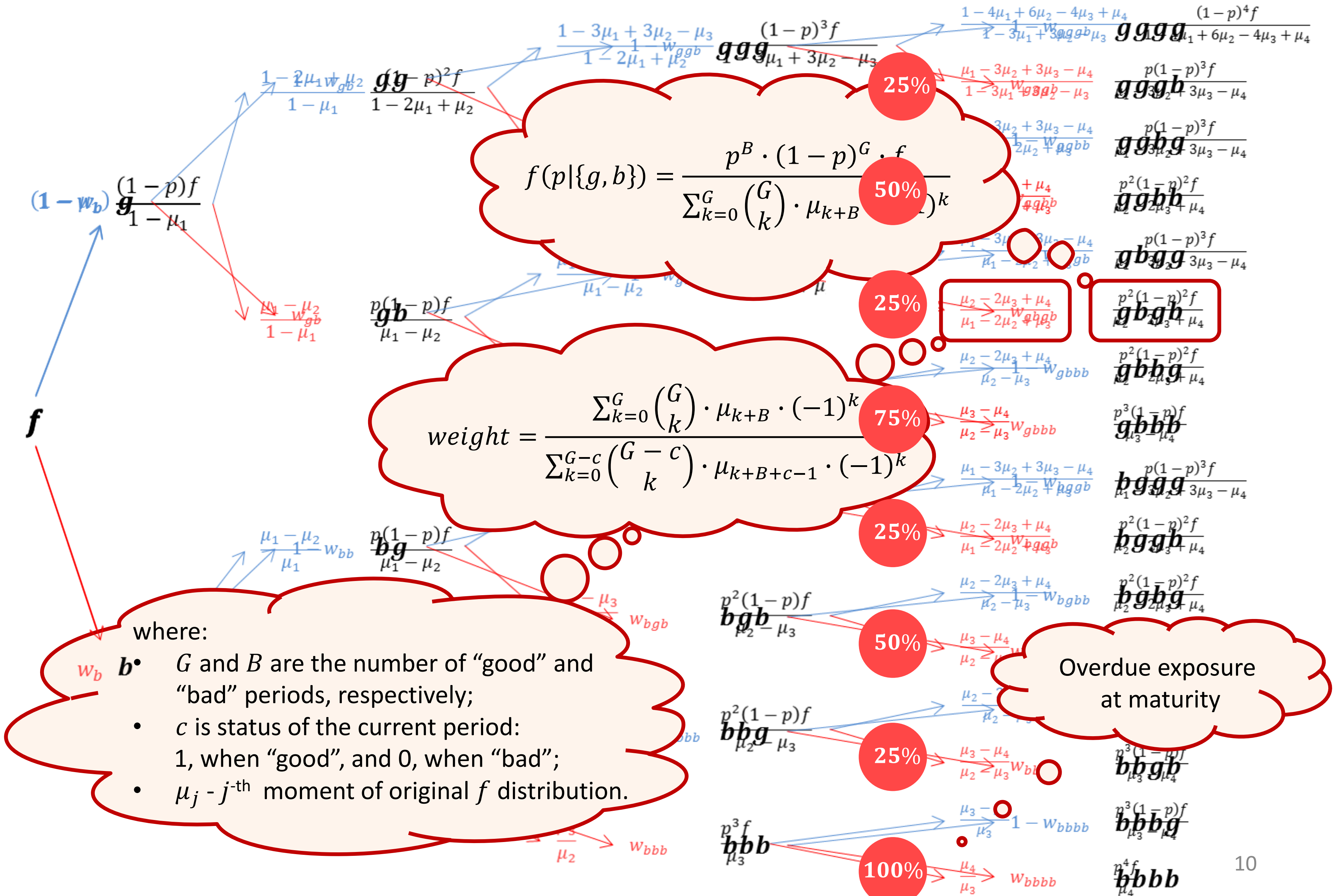
- Lets define, PD estimates a probability of **being in default status as of end of period** (1 year)

Assumptions:

1) Sufficiently large portfolio
2) Steady state (no significant portfolio growth / contraction; macroeconomic stability) $\Rightarrow f_t = f_{t+1}$
3) Recovery from default depends on the same "probability of being in default as of end of period" (i.e., $= 1 - PD_j$)
4) $\forall t \forall j \;\; \boldsymbol{E}\left(p_j^{t+1}\right) = p_j^t \qquad \Rightarrow \forall t \forall j \;\; p_j^{t+1} = p_j^t$



Life-time loss term structure



$$f(p|\{g,b\}) = \frac{p^B \cdot (1-p)^G \cdot f}{\sum_{k=0}^{G} \binom{G}{k} \cdot \mu_{k+B} \cdot (-1)^k}$$

$$weight = \frac{\sum_{k=0}^{G} \binom{G}{k} \cdot \mu_{k+B} \cdot (-1)^k}{\sum_{k=0}^{G-c} \binom{G-c}{k} \cdot \mu_{k+B+c-1} \cdot (-1)^k}$$

where:

- G and B are the number of “good” and “bad” periods, respectively;
- c is status of the current period: 1, when “good”, and 0, when “bad”;
- μ_j - j-th moment of original f distribution.

Example of Lt EL calculation

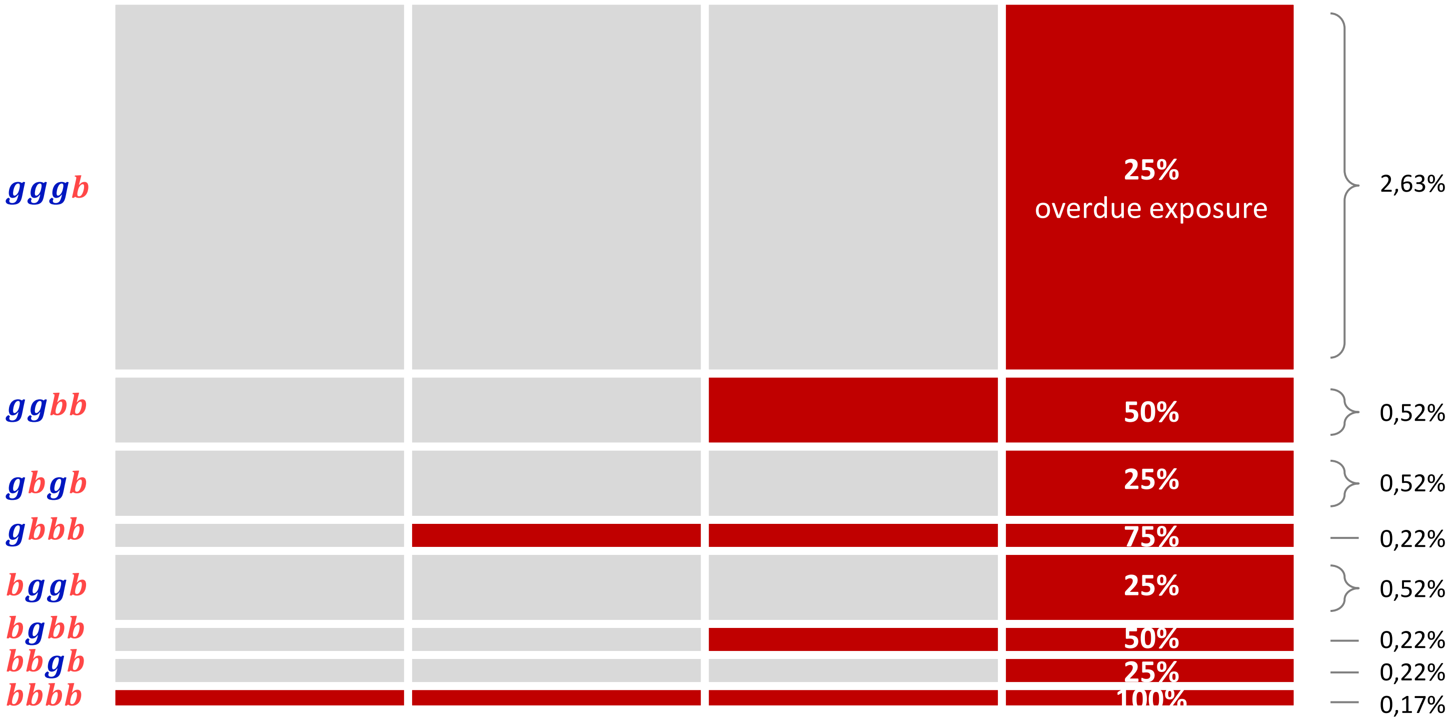
Portfolio characteristics:

- $p \sim Beta(\alpha, \beta)$
- Average PD (μ_1): 5%
- Associated Gini: 80%
- 4 years till maturity

- $\mu_0 = 1$
- $\mu_1 = 0.05$
- $\mu_2 = 0.0112$
- $\mu_3 = 0.0036$
- $\mu_4 = 0.0017$

Life-time Expected Loss:

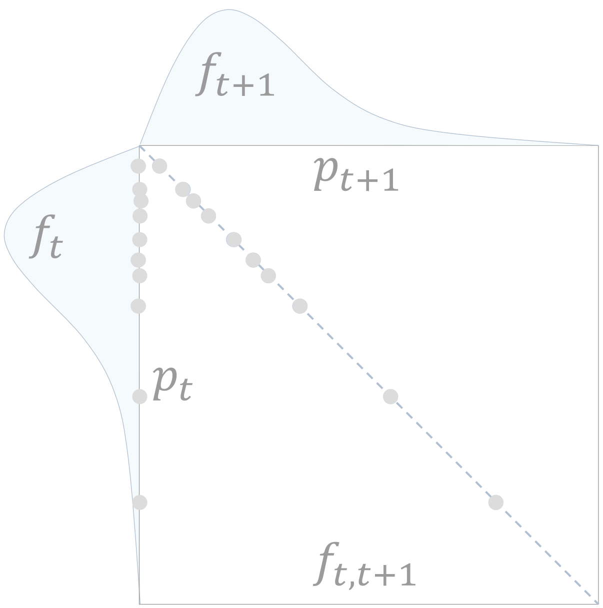
1.67%



Relaxing assumption: "individual 1-year PD is constant over time"

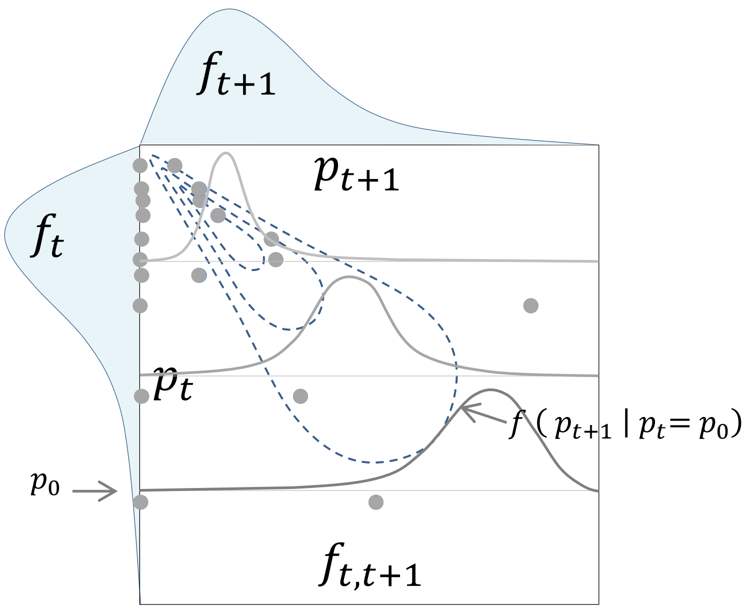
Assumptions:

- Sufficiently large portfolio
- Steady state
- $p_j^t = p_j^{t+1}$



Assumptions:

- Sufficiently large portfolio
- Steady state
- $p_{t+1} \sim f\ (p_{t+1} \mid p_t\ )$

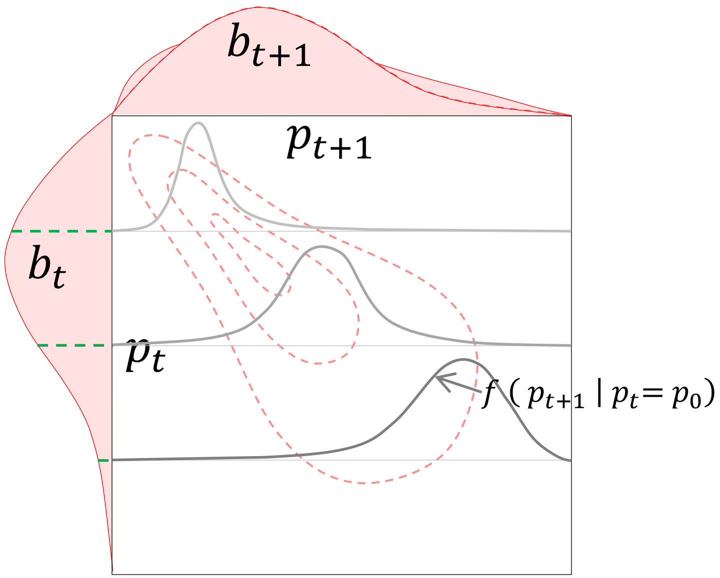


… under both sets of assumptions: $\boldsymbol{f_t = f_{t+1}}$

Relaxing assumption: “individual 1-year PD is constant over time”

Assumptions:

- Sufficiently large portfolio
- Steady state
- $p_{t+1} \sim f\,(p_{t+1} \mid p_t)$



$$b_{t+1} = \int_0^1 f(p_{t+1}|p_t) \cdot b_t \cdot dp_t$$

$$\Rightarrow \boxed{b_{t+1} = \frac{f}{\mu_1} \int_0^1 f(p_t) \cdot c(p_t, p_{t+1}) \cdot p_t \cdot dp_t}$$

Calibration

- In order to meet a specific Central Tendency level, PD can be calibrated in infinitely many ways:

$$p_1 = p_0 \times \Delta$$

$$p_1 = p_0 + \Delta$$

$$p_1 = {p_0}^2 + \Delta$$

$$\ln(DR) \sim \alpha + \beta \cdot Score$$

$$etc.$$

- Factually an offset method is based on the following assumption:

$$\frac{p_1}{1 - p_1} = \frac{p_0}{1 - p_0} \times \Delta$$

Calibration

- "Offset method" assumes:

$$E\left({}^{1}/_{X}\right) \approx {}^{1}/_{E(X)}$$

particularly for deriving the parameter "average PD": $\bar{p} = E\left(\frac{1}{1+\exp(\cdot)}\right) \approx \frac{1}{E(1+\exp(\cdot))}$

NB: If intrinsic PD really changed by this law: $\frac{p_1}{1-p_1} = \frac{p_0}{1-p_0} \times \Delta$, - then

PD distribution changed as well:

$$f_{cal} = f\left(\frac{p}{p - p\Delta + \Delta}\right)$$

And as a result, in general, Gini changed $Gini(f) \neq Gini(f_{cal})$

Expected Loss formula

- Expected Loss:

$$E(PD \cdot LGD \cdot EAD) \approx E(PD) \cdot E(LGD) \cdot E(EAD)$$

In general, there does not exist a closed-form formula for $E(X \cdot Y \cdot Z)$ (proof in footnotes)

However, we can approximate the formula very well:

$$\boxed{\begin{aligned}&E(PD \cdot LGD \cdot EAD)\\&\approx E(PD) \cdot E(LGD) \cdot E(EAD) + E(PD) \cdot Cov(LGD, EAD) + E(LGD) \cdot Cov(PD, EAD) + E(EAD) \cdot\\&Cov(LGD, PD)\end{aligned}}$$

Approximation deviates from factual EL not more than +/- 1% in 99.9% possible dependency structures.

In general, there does not exist a closed-form formula for $E(X \cdot Y \cdot Z)$. Proof:

- Lets assume each X, Y and Z can be either (-1) or 1 with probability 50%. Then, obviously $E(X \cdot Y \cdot Z) = 0$.
- Now lets define new variable $\ddot{Z} = X \cdot Y$.
- It is easy to check that $E(\ddot{Z}) = E(Z)$, $Var(\ddot{Z}) = Var(Z)$, $Cov(X, \ddot{Z}) = Cov(X, Z)$, $Cov(Y, \ddot{Z}) = Cov(Y, Z)$
- At the same time $E(X \cdot Y \cdot \ddot{Z}) = 1$

THANKS FOR YOUR INTEREST !
YOUR QUESTIONS ARE WELCOME !